Math 307Q **Final Exam A** **Winter 2018**

Your Name

Your Signature

Section (circle one) MA MB MC

Problem	Total Points	Score
1	15	
2	15	
3	20	
4	15	
5	15	
6	20	
Total	100	

- This exam is closed book. No Notes. If you forget a formula, ask one of us.
- No graphing or symbolic calculators are allowed. You may not use cell phones during the exam.
- Show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate that you have done so.
- If you are not sure what a question means, raise your hand and ask us.
- The hints are suggestions only.

1 (15 points) Find the Laplace Transform of each function:

(a) $f(t) = t^2 e^{3t}$

(b) $f(t) = \begin{cases} 0 & 0 < t < 4 \\ e^{-3t} & 4 \leq t \end{cases}$

(c) $f(t) = \begin{cases} \sin(2t) & 0 < t < \frac{\pi}{4} \\ 0 & \frac{\pi}{4} \leq t \end{cases}$

2 (15 points) Find the inverse Laplace Transforms.

(a) $\dfrac{1}{s^2 + 2s}$

(b) $\dfrac{s}{s^2 + 4s + 13}$

(c) $\dfrac{e^{-s}}{(s+1)^2 + 1}$

3 (20 points)

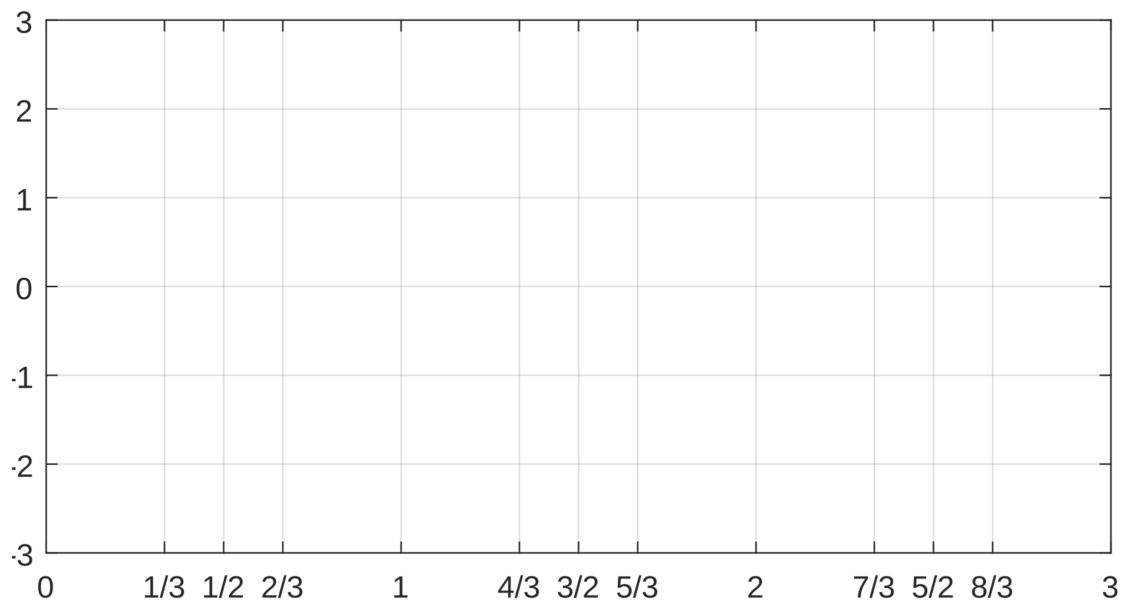
(a) Solve **using Laplace transforms**.

$$y'' + 5y' + 6y = 0$$
$$y(0) = 1 \qquad y'(0) = 2$$

(b) Find a formula for the solution as a convolution integral.

$$y'' + 5y' + 6y = e^{-3t^2}$$
$$y(0) = 0 \qquad y'(0) = 0$$

4 (15 points) Write $y(t) = \cos(2\pi t) - \cos(4\pi t)$ as a product and sketch its graph. *The positions of the grid lines have been chosen to help you draw the graph.*



Write a second order intial value problem that $y(t)$ satisfies. *I can think of two different initial value problems. Don't worry about which one you choose.Don't forget the IC's.*

5 (15 points)

(a) Compute the steady state solution to

$$mu'' + u' + u = \cos t$$

(b) Write the steady state solution in amplitude-phase form.

(c) What value of m yields the maximimu amplitude?

6 (20 points) A 1 kg bullet is fired directly upward from ground level ($y = 0$) with velocity $y'(0) = 100$ meters per second. The only forces which act are gravity $-mg$ (set $g = 10$ m/sec^2 to make the numbers easier) and air resistance $-\gamma y'(t)$.

(a) Use Newton's second law to **write a second order differential equation** for the height $y(t)$. Add initial conditions. To make the calculations below easier, change the notation by replacing γ with $\frac{1}{\tau}$. *Hint: If it helps, think of this as a forced damped mass-spring system without the spring (i.e. $k = 0$).*

(b) Compute the solution to the initial value problem as a function of t and τ.

(continued on next page)

(Problem 6 continued)

(c) The bullet lands 15 seconds later. Write down an equation for the coefficient τ.

(d) Make the assumption that $\frac{15}{\tau}$ is big enough that you can ignore the term $e^{\frac{-15}{\tau}}$. Now solve for τ.

Table of Laplace Transforms

1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$u_a(t)$	$\frac{e^{-as}}{s}$
$\delta_a(t)$	e^{-as}
$y'(t)$	$sY(s) - y(0)$
$y''(t)$	$s^2Y(s) - y(0)s - y'(0)$
$e^{at}y(t)$	$Y(s-a)$
$ty(t)$	$-\frac{d}{ds}Y(s)$
$u_a(t)y(t-a)$	$e^{-as}Y(s)$
$u_a(t)y(t)$	$e^{-as}\mathcal{L}\{y(t+a)\}$
$y(at)$	$\frac{1}{a}Y(\frac{s}{a})$

Trig Formulas

$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$
$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$
$$\cos\theta\cos\phi = \frac{1}{2}\left(\cos(\theta+\phi) + \cos(\theta-\phi)\right)$$
$$\sin\theta\sin\phi = \frac{1}{2}\left(\cos(\theta-\phi) - \cos(\theta+\phi)\right)$$
$$\sin\theta\cos\phi = \frac{1}{2}\left(\sin(\theta+\phi) + \sin(\theta-\phi)\right)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$
$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$$
$$\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$$
$$\cosh\theta = \frac{e^{\theta}+e^{-\theta}}{2}$$
$$\sinh\theta = \frac{e^{\theta}-e^{-\theta}}{2}$$