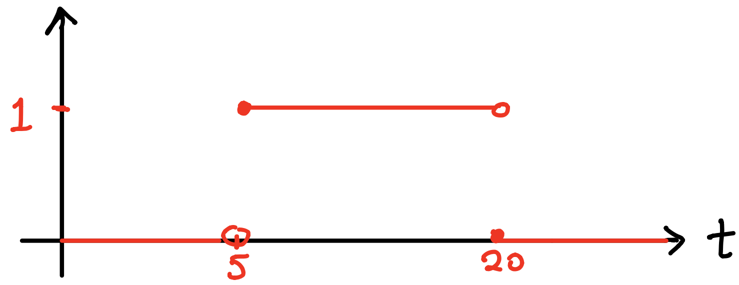
$u_5(t) - u_{20}(t) =$



<u>Ex</u> Solve $2y'' + y' + 2y = u_5(t) - u_{20}(t)$

$$y(0) = 0, \quad y'(0) = 0$$

$$\mathcal{L}\{2y'' + y' + 2y\} = \mathcal{L}\{u_5(t) - u_{20}(t)\}$$

$$2\mathcal{L}\{y''\} + \mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = \mathcal{L}\{u_5(t)\} - \mathcal{L}\{u_{20}(t)\}$$

$$2\left(s^2 Y(s) - s y(0) - y'(0)\right) + \left(sY(s) - y(0)\right) + 2Y(s) = \frac{e^{-5s}}{s} - \frac{e^{-20s}}{s}$$

$$(2s^2 + s + 2)\, Y(s) = \frac{1}{s}\left(e^{-5s} - e^{-20s}\right)$$

$$Y(s) = \left(e^{-5s} - e^{-20s}\right) H(s)$$

$$H(s) = \frac{1}{s(2s^2 + s + 2)}$$

$$h(t) = \mathcal{L}^{-1}\{H(s)\}$$

$$\boxed{y(t) = \mathcal{L}^{-1}\{Y(s)\} = u_5(t)\, h(t-5) - u_5(t)\, h(t-20)}$$

<u>Find h</u> :

Partial fraction

$$H(s) = \frac{A}{s} + \frac{Bs + C}{2s^2 + s + 2} = \frac{A(2s^2 + s + 2) + (Bs + C)s}{s(2s^2 + s + 2)}$$

$$= \frac{(2A + B)s^2 + (A + C)s + 2A}{s(2s^2 + s + 2)}$$

$$(2A+B)s^2 + (A+C)s + 2A = 1$$

$$\left\{\begin{array}{l} 2A+B=0 \\ A+C=0 \\ 2A=1 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} A=\frac{1}{2} \\ B=-1 \\ C=-\frac{1}{2} \end{array}\right\}$$

$$H(s) = \frac{1/2}{s} - \frac{s+1/2}{2s^2+s+2}$$

$$= \frac{1/2}{s} - \left(\frac{1}{2}\right)\frac{s+1/2}{s^2+\frac{1}{2}s+1}$$

$$= \frac{1/2}{s} - \frac{1}{2}\left(\frac{s+\frac{1}{4}}{(s+\frac{1}{4})^2+\left(\frac{\sqrt{15}}{4}\right)^2} + \frac{1}{\sqrt{15}}\frac{\frac{\sqrt{15}}{4}}{(s+\frac{1}{4})^2+\left(\frac{\sqrt{15}}{4}\right)^2}\right)$$

$$\boxed{h(t) = \frac{1}{2} - \frac{1}{2}\left[e^{-\frac{t}{4}}\cos\left(\frac{\sqrt{15}}{4}t\right) + \frac{1}{\sqrt{15}}e^{-\frac{t}{4}}\sin\left(\frac{\sqrt{15}}{4}t\right)\right]}$$

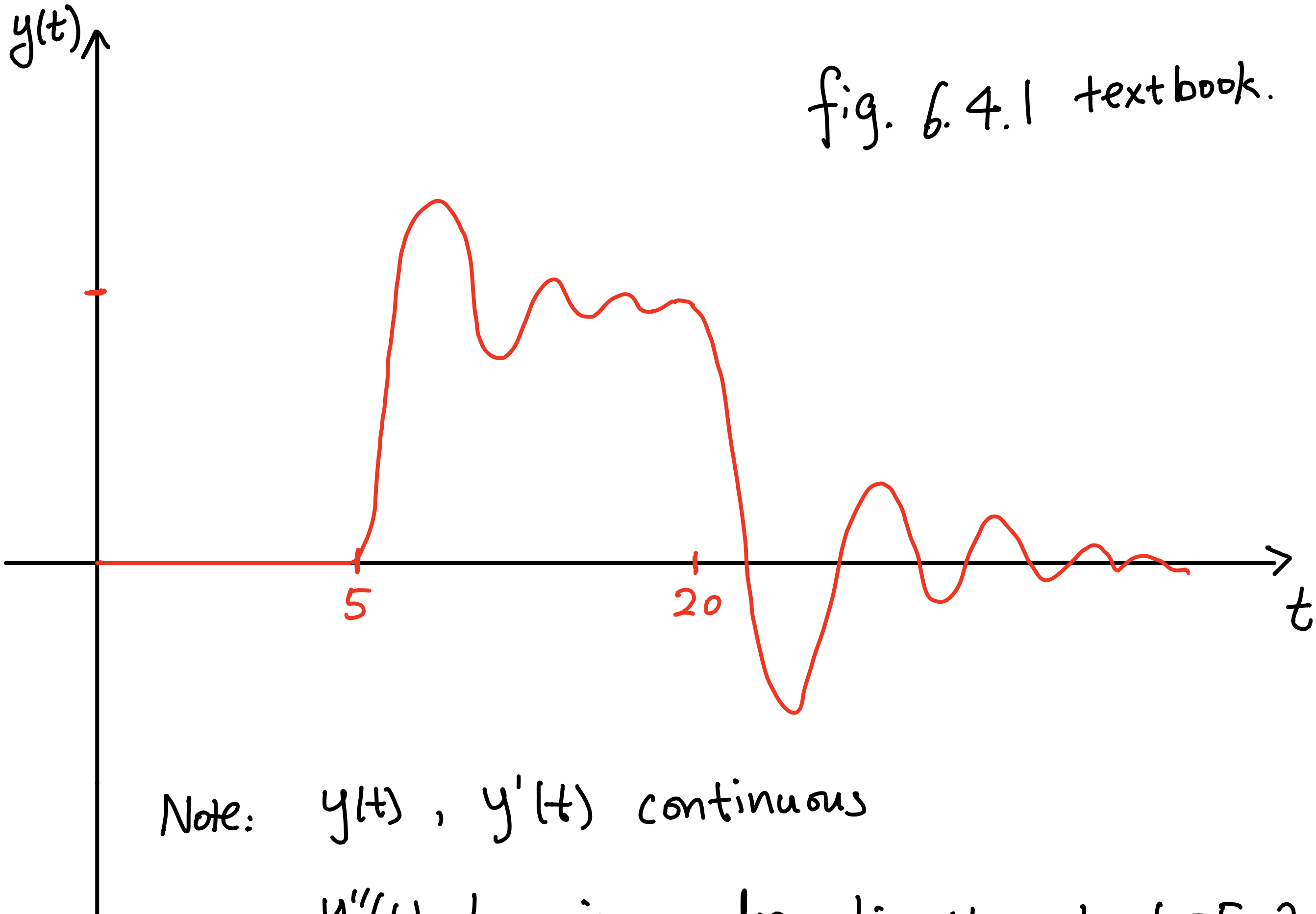


fig. 6.4.1 textbook.

Note: $y(t)$, $y'(t)$ continuous

$y''(t)$ has jump discontinuity at $t=5, 20$.

Table of Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}$, $s > 0$
2. e^{at}	$\frac{1}{s-a}$, $s > a$
3. $\sinh at = \frac{e^{at}-e^{-at}}{2}$	$\frac{a}{s^2-a^2}$, $s > \|a\|$
4. $\cosh at = \frac{e^{at}+e^{-at}}{2}$	$\frac{s}{s^2-a^2}$, $s > \|a\|$
5. t^n, $n =$positive integer	$\frac{n!}{s^{n+1}}$, $s > 0$
6. $t^n e^{at}$, $n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}$, $s > a$
7. $\sin bt$	$\frac{b}{s^2+b^2}$, $s > 0$
8. $\cos bt$	$\frac{s}{s^2+b^2}$, $s > 0$
9. $e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$, $s > a$
10. $e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$, $s > a$
11. $u_c(t)$	$\frac{e^{-cs}}{s}$, $s > 0$
12. $u_c(t)f(t-c)$	$e^{-cs}F(s)$
13. $e^{ct}f(t)$	$F(s-c)$
~~14. $\delta(t-c)$~~	~~e^{-cs} when $c \geq 0$, 0 when $c < 0$~~
15. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
16. $(-t)^n f(t)$	$F^{(n)}(s)$
~~17. $\int_0^t f(t-\tau)g(\tau)d\tau$~~	~~$F(s)G(s)$~~

$$\frac{s+\frac{1}{2}}{s^2+\frac{1}{2}s+1} = \frac{s+\frac{1}{2}}{(s+\frac{1}{4})^2+\left(\frac{\sqrt{15}}{4}\right)^2}$$

$$= \frac{s+\frac{1}{4}}{(s+\frac{1}{4})^2+\left(\frac{\sqrt{15}}{4}\right)^2} + \frac{\frac{1}{4}}{(s+\frac{1}{4})^2+\left(\frac{\sqrt{15}}{4}\right)^2}$$

$$= \frac{s+\frac{1}{4}}{(s+\frac{1}{4})^2+\left(\frac{\sqrt{15}}{4}\right)^2} + \frac{1}{\sqrt{15}}\,\frac{\frac{\sqrt{15}}{4}}{(s+\frac{1}{4})^2+\left(\frac{\sqrt{15}}{4}\right)^2}$$

$$= \mathcal{L}\left\{e^{-\frac{t}{4}}\cos\left(\frac{\sqrt{15}}{4}t\right)\right\} + \frac{1}{\sqrt{15}}\mathcal{L}\left\{e^{-\frac{t}{4}}\sin\left(\frac{\sqrt{15}}{4}t\right)\right\}$$

$a = -\frac{1}{4}$

$b = \frac{\sqrt{15}}{4}$

$$\frac{\frac{1}{4}}{b}\,\frac{b}{(\quad)}$$

$$\frac{\frac{1}{4}}{b} = \frac{\frac{1}{4}}{\frac{\sqrt{15}}{4}} = \frac{1}{\sqrt{15}}$$

$$\frac{s+\frac{1}{2}}{s^2+\frac{1}{2}s+\boxed{1}}$$

$$= \frac{s+\frac{1}{2}}{(s+\frac{1}{4})^2+\boxed{\frac{15}{16}}}$$

$$s^2+\frac{1}{2}s+\boxed{\frac{1}{16}}$$