Math 308 Midterm #1, Autumn 2017

Name:

ID#:

Signature:

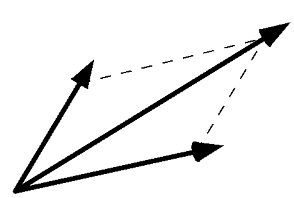
All work on this exam is my own.

Instructions.

- You are allowed a calculator and notesheet (handwritten, two-sided). Hand in your notesheet with your exam.
- Other notes, devices, etc are not allowed.
- Unless the problem says otherwise, **show your work** (including row operations if you row-reduce a matrix) and/or **explain your reasoning**. You may refer to any theorems, facts, etc, from class.
- All the questions can be solved using (at most) simple arithmetic. (If you find yourself doing complicated calculations, there might be an easier solution...)

1	/20
2	/20
3	/20
4	/10
5	/20

Good luck!



(1) (a) [5 pts each] Compute:

$$3\begin{bmatrix}1\\2\\1\end{bmatrix} - \frac{1}{2}\begin{bmatrix}0\\2\\2\end{bmatrix} =$$

$$\begin{bmatrix}0 & 1 & -1\\2 & 3 & 0\end{bmatrix} \cdot \begin{bmatrix}1\\1\\-2\end{bmatrix} =$$

(b) [10 pts] Determine the general solution to the following system of equations:

$$\begin{aligned} z_1 + z_2 + 2z_3 \phantom{{}+z_4} &= 0 \\ 2z_1 + 2z_2 + 5z_3 + z_4 &= 1 \end{aligned}$$

Express your answer in vector form.

(2) (a) [10 pts] Does this set of vectors span $\mathbb{R}^3$?

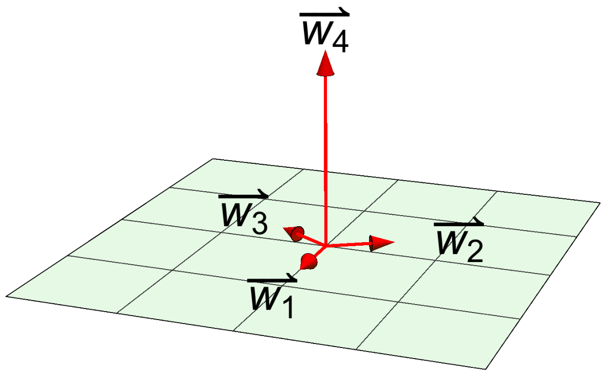
$$\vec{\mathbf{v}}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{\mathbf{v}}_2 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \quad \vec{\mathbf{v}}_3 = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}.$$

(b) [10 pts] Does this set of vectors span $\mathbb{R}^5$?

$$\vec{\mathbf{u}}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 4 \\ -2 \end{bmatrix}, \quad \vec{\mathbf{u}}_2 = \begin{bmatrix} 0 \\ 2 \\ 5 \\ 0 \\ -3 \end{bmatrix}, \quad \vec{\mathbf{u}}_3 = \begin{bmatrix} -1 \\ 0 \\ 2 \\ -2 \\ 1 \end{bmatrix}.$$

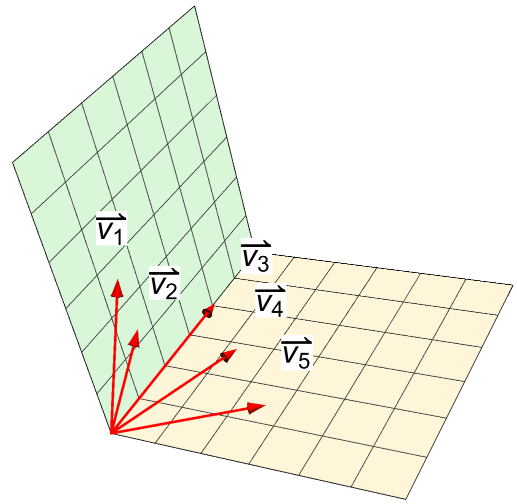
(3) Consider these arrangements of vectors in $\mathbb{R}^3$, then answer the questions below. **No justification is necessary.**

(I)



(Note: The plane contains $\vec{\mathbf{w}}_1, \vec{\mathbf{w}}_2, \vec{\mathbf{w}}_3$.)

(II)



(Note: The left plane contains $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3$. The right plane contains $\vec{\mathbf{v}}_3, \vec{\mathbf{v}}_4, \vec{\mathbf{v}}_5$.)

[6 pts] (a) From (I), with the vectors $\vec{\mathbf{w}}_1, \vec{\mathbf{w}}_2, \vec{\mathbf{w}}_3, \vec{\mathbf{w}}_4$:

Give any set of linearly independent vectors: $\left\{ \qquad\qquad\qquad \right\}$

Give any set of linearly dependent vectors: $\left\{ \qquad\qquad\qquad \right\}$

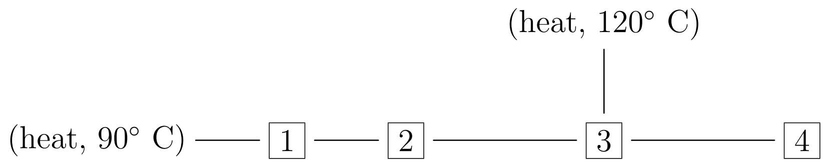
[8 pts] (b) From (II): Which of the following sets are linearly independent? Circle them:

$\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$ $\qquad$ $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_3, \vec{\mathbf{v}}_5\}$ $\qquad$ $\{\vec{\mathbf{v}}_3, \vec{\mathbf{v}}_4\}$ $\qquad$ $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_4, \vec{\mathbf{v}}_5\}$

[6 pts] (c) Consider the 3×2 matrix $A = [\ \vec{\mathbf{w}}_1 \mid \vec{\mathbf{w}}_2\ ]$. Which of the following equations have solutions $\vec{\mathbf{x}}$? Circle them:

$A\vec{\mathbf{x}} = \vec{\mathbf{w}}_3$ $\qquad\qquad$ $A\vec{\mathbf{x}} = \vec{\mathbf{w}}_4$ $\qquad\qquad$ $A\vec{\mathbf{x}} = \vec{\mathbf{0}}$

(4) A chemical factory (Levinson's Linear Laboratory) has four tanks of liquid connected in a line, along with two heaters:



After a long time, the temperature t_i of the i-th tank will be the *average* temperature of the tanks and heaters connected to it. For example, t_1 should be the average of t_2 and 90.

(a) [5 pts] Write the system of equations you would use to determine t_1, t_2, t_3, t_4. You do **not** need to solve.

(b) [5 pts] Write the corresponding augmented matrix. You do **not** need to solve.

(5) In each of the following, either give an example or write "impossible".
No justification is necessary. [5 pts each]

(a) A set of vectors that spans $\mathbb{R}^2$ and is linearly dependent.

(b) A set of 4 vectors in $\mathbb{R}^3$ that do not span $\mathbb{R}^3$.

(c) Three vectors that span $\mathbb{R}^3$ and satisfy the equation $\vec{\mathbf{v}}_1 - 2\vec{\mathbf{v}}_2 + \vec{\mathbf{v}}_3 = \vec{\mathbf{0}}$.

(d) An echelon system of equations in variables x_1, x_2, x_3 with free variable x_3. (Write out the equations.)

[**2 pts**] **Bonus.** What have you found easiest and hardest in Math 308?

Do you wish the pace was (circle): FASTER   ABOUT THE SAME   SLOWER

(OR: If you don't want to answer, draw a picture involving vectors.)