Your Name

Your Signature

Student ID #

<u>Honor Statement</u>

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: ______________________________

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 6 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	20	
2	10	
3	10	
4	10	
Spicy Bonus	6	
Total	50	

1. (20 points) Indicate whether the given statement is true or false (2 pts) and give justification as to why it is true or false(2 pts).

   a) [4 pts] Let $\{\vec{\mathbf{u}}_1, \vec{\mathbf{u}}_2, \vec{\mathbf{u}}_3\}$ be a linearly independent set in $\mathbb{R}^3$, then the set $\{c\vec{\mathbf{u}}_1, c\vec{\mathbf{u}}_2, c\vec{\mathbf{u}}_3\}$ is linearly independent, for all scalars $c \neq 0$.

   b) [4 pts] There exists a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}$ such that

$$T\left(\begin{bmatrix}1\\0\\1\end{bmatrix}\right) = 3 \quad \text{and} \quad T\left(\begin{bmatrix}4\\0\\4\end{bmatrix}\right) = 9$$

   c) [4 pts] If $\vec{\mathbf{u}}_1, \vec{\mathbf{u}}_2$, and $\vec{\mathbf{v}}$ are vectors in $\mathbb{R}^3$, and $\vec{\mathbf{v}}$ is in span$\{\vec{\mathbf{u}}_1, \vec{\mathbf{u}}_2\}$, then $\{\vec{\mathbf{u}}_1, \vec{\mathbf{u}}_2, \vec{\mathbf{v}}\}$ can never span $\mathbb{R}^3$.

Give an example of each of the following. If it is not possible write "NOT POSSIBLE". No justification is needed if it is not possible.

d) [2 pt] A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, given by $T(\vec{\mathbf{x}}) = A\vec{\mathbf{x}}$, which reflects the unit square about the x-axis. (Note: Take the unit square to lie in the first quadrant. Giving the matrix of T, if it exists, is a sufficient answer).

e) [2 pt] A set of 4 vectors in $\mathbb{R}^3$ that spans $\mathbb{R}^3$ and is linearly **dependent**.

f) [2 pt] 3 linearly independent vectors $\vec{\mathbf{u}}_1, \vec{\mathbf{u}}_2, \vec{\mathbf{u}}_3$ satisfying the equation $2\vec{\mathbf{u}}_1 + 3\vec{\mathbf{u}}_2 - 4\vec{\mathbf{u}}_3 = \vec{\mathbf{0}}$.

g) [2 pts] A **homogeneous** linear system with strictly more variables than equations, having exactly one solution.

2. (10 points) Consider the following linear system with a and b nonzero constants.

$$\begin{cases} x_1 - 3x_2 + x_3 = 4 \\ 2x_1 - 8x_3 = -2 \\ -6x_1 + 6x_2 + ax_3 = b \end{cases}$$

a) [5 pts] For what values of a and b does the system have infinitely many solutions?

a) [5 pts] Give an example of a and b where the system has exactly one solution.

3. (10 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be given by $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -x_1 + 3x_2 + 2x_3 \\ 4x_1 - 12x_2 - 8x_3 \end{bmatrix}$.

a) [3pts] Determine the matrix associated to T. That is, find the matrix A such that $T(\vec{\mathbf{x}}) = A\vec{\mathbf{x}}$.

b) [3 pts] Is T one-to one? Explain your answer.

c) [4pts] Is T onto? If not, find a vector not in the range of T.

4. (10 points) a) [4 pts] Let $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \mathbf{u}_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$. Show that the set $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ spans $\mathbb{R}^3$.

b) [4 pts] Express the vector $\mathbf{v} = \begin{bmatrix} 3 \\ 5 \\ 11 \end{bmatrix}$ as a linear combination of the vectors from the previous part.

c) [2 pts] Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that

$$T(\mathbf{u}_1) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, T(\mathbf{u}_2) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, T(\mathbf{u}_3) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Compute $T(\mathbf{v})$ (where $\mathbf{v}$ is the same vector as in part b).

Super Spicy Bonus Question

5. (6 points) a) [4pts] Let $f(x) = ax^2 + bx + c$ denote an arbitrary polynomial of degree 2, with constants a, b, and c. To each such polynomial, associate the vector

$$ax^2 + bx + c \leftrightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Let T be the linear transformation given by $T(f(x)) = f'(x)$. In other words, T is the linear transformation that eats a degree 2 polynomial and spits out its derivative (Yes it is linear!). Find the matrix corresponding to T. (Hint: Remember that if we are given a linear transformation T as above, its matrix is completely determined by $T(\mathbf{e}_i)$, i.e. $T(\mathbf{x}) = A\mathbf{x}$ where $A = [T(\mathbf{e}_1)\ \ T(\mathbf{e}_2)\ \ T(\mathbf{e}_3)]$

b) [2 pts] Is T one-to one? Is it onto? Explain your answer.