Math 208 Final

June 9, 2025

NAME:

UW EMAIL:

STUDENT ID NUMBER:

1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
7	/10
8	/10
9	/10
10	/10
Bonus	/2
Total	/100

Instructions. [1]

- For each problem below, give a carefully explained solution using the vocabulary and notation from class. A correct answer with no supporting work or explanation will receive at most 1 point.
- Put a $\boxed{\text{box}}$ around your final answers.
- You are allowed a simple calculator and notesheet. Other notes, electronic devices, etc, are not allowed.
- You must keep your dominant writing hand on your desk at all times, including while using your calculator.
- Remain seated until all exams have been collected and you are dismissed. Please stay in your seat until all exams are collected and the class is dismissed.

Good luck!

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[1] Test code: 2321

(1) Compute the reduced row-echelon form (RREF) of the matrix A below by doing one elementary row operation at a time. Box your final RREF.

$$A = \begin{pmatrix} 2 & 0 & 8 & 0 & 4 \\ -1 & 1 & -1 & 1 & x \\ 0 & 1 & 3 & 2 & x+2 \end{pmatrix}$$

(2) Decide whether the two vectors

$$\vec{v} = \begin{pmatrix} 1 \\ 3 \\ 2 \\ 0 \end{pmatrix} \text{ and } \vec{w} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 2 \end{pmatrix}.$$

can be extended to a basis of $\mathbb{R}^4$.

- If yes, construct a full basis that includes $\vec{v}$ and $\vec{w}$, and justify that it is linearly independent.
- If not, explain why not.

(3) Determine if the matrix $B = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ is invertible or not.

- If yes, find B^{-1}.
- If no, justify why B does not have an inverse.

(4) What are the eigenvalues of the following matrix?

$$\begin{pmatrix} -1 & 6 & 2 & 0 & 12 \\ 0 & -41 & 3 & 12 & 2 \\ 0 & 0 & 3 & 1 & 6 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 2 & -1 \end{pmatrix}$$

(5) Let

$$D = \begin{bmatrix} 1 & 1 \\ 3 & 0 \\ 0 & 3 \\ 0 & 6 \end{bmatrix}.$$

Find a basis for the subspace S of all vectors in $\mathbb{R}^4$ that are orthogonal to the column span of D.

(Hint: two vectors are orthogonal if their dot product is 0.)

(6) What is the area of the triangle with vertices $(0, 9), (5, 2), (8, 6)$?

(7) Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be given by

$$T(w, x, y, z) = \begin{bmatrix} 5w - x \\ 2w \\ 7w + 2x + y \\ 13w + 41x + 11y + 4z \end{bmatrix}.$$

What is the dimension of the kernel of T? Show your work.

(8) Say $G = ABA^{-1}$ where

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 3 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Find the eigenvalues of G and give a basis for each of their associated eigenspaces. Justify your answer.

(9) Let

$$F = \begin{pmatrix} 2 & 5 & 1 \\ 0 & -3 & -1 \\ 2 & 14 & 4 \end{pmatrix}.$$

The matrix F has characteristic polynomial $-\lambda(\lambda - 1)(\lambda - 2)$. Find a basis for the eigenspaces of A corresponding to each of its eigenvalues.

(10) Let $A = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

Describe geometrically the linear map $T\vec{x} = AB\vec{x}$ acting on the plane (e.g. reflections, shears, scalings). Then sketch the Knill smiley face under the transformation T assuming the origin is at the center of the face, and state the key features justifying your sketch.

Bonus (2 points) Let

$$A(x) = \begin{pmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{pmatrix}.$$

Find all real values of x for which $A(x)$ is invertible and compute a simple closed formula for it's inverse when it exists.

This page is intentionally left blank for scratch work or designing a cartoon about a Math 208 concept if you have extra time.