Name____

Tuesday meeting time with Collin (12:30, 1:30, or 2:30)____________

Math 308P Final Exam Wednesday, December 12, 2018, 2:30–4:20

Six problems for a total of 100 points. Please read each problem carefully before answering.

1. (24 points – 6 points each) Find examples of

(a) a linear transformation $T_1 : \mathbb{R}^2 \longrightarrow \mathbb{R}^4$ and a linear transformation $T_2 : \mathbb{R}^4 \longrightarrow \mathbb{R}^2$ such that the transformation $T(\overline{x}) = T_2(T_1(\overline{x}))$ is both one-to-one and onto;

(b) a 3×3 matrix with characteristic polynomial $(1 - \lambda)^3$ such that the eigenspace for $\lambda = 1$ is only 1-dimensional;

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(c) a 4×4 matrix with no real eigenvalues;

(d) a 3×5 reduced echelon matrix that has the most possible nonzero entries.

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2. (12 points) Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a matrix with determinant equal to 5. Let

$$A = \begin{bmatrix} a & b & 0 \\ c & d & 0 \end{bmatrix} \qquad \textbf{and} \qquad B = \begin{bmatrix} d & -b \\ -c & a \\ 0 & 0 \end{bmatrix}.$$

Find AB and BA. **Be sure to label your answers (which is AB and which is BA). Your answers should be matrices with numerical entries, not letters.**

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3. (12 points) Let $T(\overline{x}) = A\overline{x}$, where $A = \begin{bmatrix} 1 & 0 & 2 & -1 \\ 1 & 3 & -1 & 5 \\ 2 & 3 & 1 & 4 \end{bmatrix}$. Find

(a) a basis for the kernel of T, and

(b) a basis for the range of T. Please show your work clearly.

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4. (12 points) Find all values of x for which the matrix $\begin{bmatrix} 2 & 1 & 0 \\ 6 & x & x \\ x & 0 & 2 \end{bmatrix}$ is not invertible. Please show your work clearly.

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5. (16 points) Find a 2×2 matrix that has eigenvector $\begin{bmatrix} 7 \\ 3 \end{bmatrix}$ with eigenvalue 3 and eigenvector $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ with eigenvalue 2. Please show your work clearly.

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6. (24 points) Recall that $\sinh(t) = (e^t - e^{-t})/2$ and $\cosh(t) = (e^t + e^{-t})/2$. Let $f(t)$ be a function of the form $x_1 \sin(t)\sinh(t) + x_2 \sin(t)\cosh(t) + x_3 \cos(t)\sinh(t) + x_4 \cos(t)\cosh(t)$, where x_1, x_2, x_3, x_4 are coefficients. The derivative will have the same form, so we can write $f'(t) = y_1 \sin(t)\sinh(t) + y_2 \sin(t)\cosh(t) + y_3 \cos(t)\sinh(t) + y_4 \cos(t)\cosh(t)$. Please show all your steps clearly.

(a) Find the matrix A of the transformation from $\overline{x}$ to $\overline{y}$.

(b) Find A^{-1}.

(c) **Using part (b)**, find

$$\int (4\sin(t)\sinh(t) + 10\cos(t)\cosh(t))dt.$$

ANSWERS

1. (a) For example, take $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ for T_2 and A^T for T_1.

(b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is an example.

(c) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ is an example.

(d) $\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$ is an example (with 9 nonzero entries).

2. $AB = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ and $BA = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

3. The reduced echelon form is $\begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. (a) $[-2\ 1\ 1\ 0]^T$, $[1\ -2\ 0\ 1]^T$.

(b) $[1\ 1\ 2]^T$, $[0\ 3\ 3]^T$.

4. Using the cofactor expansion of the determinant along the first row, we get $4x + x^2 - 12$, which we set equal to zero to find out when the matrix is not invertible. The quadratic equations gives $x = -2 \pm \sqrt{16}$, that is, x is 2 or -6.

5. $PDP^{-1} = \begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ -3/2 & 7/2 \end{bmatrix} = \begin{bmatrix} 9 & -14 \\ 3 & -4 \end{bmatrix}$.

6. (a) $A = \begin{bmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$. (b) $\begin{bmatrix} 0 & 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ -1/2 & 0 & 0 & 1/2 \\ 0 & -1/2 & 1/2 & 0 \end{bmatrix}$.

(c) 4 times the first column of A^2 plus 10 times the last column leads to $7\sin(t)\cosh(t) + 3\cos(t)\sinh(t) + C$.