

MATH 207 O  
MIDTERM 2  
AUTUMN 2021

Name \_\_\_\_\_ Student ID # \_\_\_\_\_

HONOR STATEMENT

“I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.”

SIGNATURE: \_\_\_\_\_

1	10	
2	10	
3	10	
4	8	
5	12	
Total	50	

- Pace yourself. You have 50 minutes to complete the exam.
- Do not write within 1 cm of the edges of each page as these exams will be scanned.
- If you need more space, you may use the back of the page but be sure to clearly indicate that you have done so.
- Unless otherwise indicated, show all your work and justify your answers.
- You may use the Texas Instruments TI-30X IIS calculator and a 8.5×11-inch double-sided sheet of handwritten notes. All other electronic devices (including graphing and programmable calculators and calculators with calculus functions) are not permitted.
- The use of headphones or earbuds during the exam is not permitted.
- You are not allowed to use your phone for any reason during this exam. Turn your phone off and put it away for the duration of the exam.
- Raise your hand if you have any questions.

GOOD LUCK!

1. (a) (6 points) Find the general solution of the differential equation:

$$y'' - 10y' + 29y = 0$$

- (b) (4 points) Find the unique solution that satisfies the initial conditions:

$$y(0) = \frac{5}{2}, \quad y'(0) = \frac{1}{2}$$

2. (a) (4 points) Find a particular solution to

$$y'' - y' - 2y = 5 \sin(2t)$$

- (b) (4 points) Find a particular solution to

$$y'' - y' - 2y = 3e^t - e^{-t}$$

- (c) (2 points) Give the general solution to

$$y'' - y' - 2y = 5 \sin(2t) + 3e^t - e^{-t}$$

3. (a) (6 points) A spring-mass system has the following equation of motion:

$$u(t) = -4 \cos(5t) + 4 \sin(5t), \quad t \geq 0.$$

- i. Rewrite  $u(t)$  in the form  $R \cos(\omega t - \varphi)$ .

- ii. Find the first time the mass passes through its equilibrium position.

- (b) (4 points) A spring-mass system with external forcing function having the equation

$$3u'' + ku = 7 \cos(4t)$$

experiences resonance. Find the value of  $k$  and briefly explain the main characteristics the resulting motion (max two sentences).

4. (8 points) Find a second order differential equation whose general solution is

$$y = c_1 e^{3t} + c_2 t e^{3t} + e^{-2t} + t^2 - 3$$

5. A certain spring is known to stretch 0.06 m beyond its natural length when a force of 0.54 N is applied to it. An unknown mass  $m$  is attached to this spring along with a damping device with constant  $\gamma = 6 \text{ N} \cdot \text{sec}/\text{m}$ . The mass is pushed up 0.04 m and released from rest. The resulting motion is observed to oscillate with decreasing amplitude and a quasi-period of  $T = 4\pi/3$  sec. Use the convention that the downward direction is positive.

(a) (8 points) Find the value of  $m$ . (Hint: What is the quasi-frequency?)

(b) (4 points) Write down the initial value problem for the above spring-mass system (do not solve).