

Name (PRINT): _____

Student ID (PRINT): _____

Autumn 2022 – Introduction to Differential Equations Second Examination

Instructions

1. The use of all electronic devices is prohibited. Any electronic device needs to be turned off and placed in your bag. Any textbooks or notes also need to be placed in your bag.
2. Present your solutions in the space provided. Show all your work neatly and concisely. Clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

Scholastic dishonesty will not be tolerated and may result in terminating the midterm early. The work on this test is my own.

Signature: _____

Exercise 1. (5 points)

- Find the solution to the initial value problem

$$y'' - y = 0, \quad y(0) = 5, \quad y'(0) = -3$$

- Find the minimal value of the solution.

Exercise 2. (5 points) Consider the initial value problem

$$y'' + 2y' + 5y = 0, \quad y(0) = 2, \quad y'(0) = \alpha > 0$$

- Find the solution to the initial value problem.
- Find α such that $y = 0$ when $x = 1$.

Leave your answer using usual function in the form $\alpha = \dots$. No numerical approximation is required.

$\alpha = e^{-25 \frac{\text{Arctan}17}{\ln(\pi)}}$ is a valid format for your answer.

Exercise 3. (5 points) Given the differential equation

$$y'' + 2y' + y = xe^{-x}$$

- Make a valid guess for the particular solution.
- Find the general solution

Exercise 4. (5 points) Make a valid guess for a particular solution if the method of undetermined coefficient is to be used:

$$y'' + 4y = e^{-2x} + x^2 + \sin 3x + 5xe^{2x} + x^2 \cos 2x - 3xe^{2x} \sin 2x$$

Exercise 5. (5 points) A spring with a mass of 2 kg has a damping constant 8 Ns/m (Newton * Second/Meter). A force of 3 N is required to stretch the spring 0.5 m beyond its natural length.

The spring is initially stretched 1 m beyond its natural length and released with no initial velocity.

- Find the position $u(t)$ as a function of the time.
- Find the mass that needs to be attached to the spring to produce critical damping.