MATH 207G: MIDTERM 2

Name: _____

Please do **not** start working until given the indication. You have 50 minutes for the exam, which has 5 problems. There is an extra page at the end for scratch work if necessary. Good luck!

Q1 (10 points)

(a) Find the general solution to $y'' + 2y' + y = 0$.

(b) Write down a linear, homogenenous 2nd order differential equation with constant coefficients that has $y(t) = C_1 \cos 4t + C_2 \sin 4t$ as its general solution.

(c) Find the **real part** of the complex function $f(t) = e^{(2+3i)t}$.

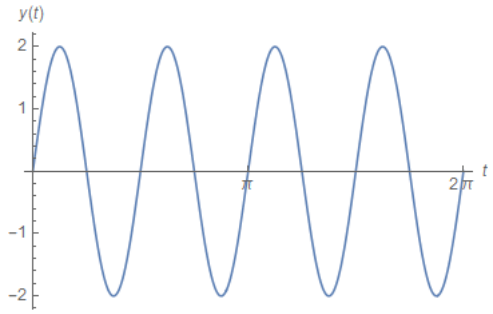
Q2 (10 points)

Solve the initial value problem

$$y'' - y = \cos 2t, \qquad y(0) = 1, \quad y'(0) = 0.$$

Q3 (10 points)

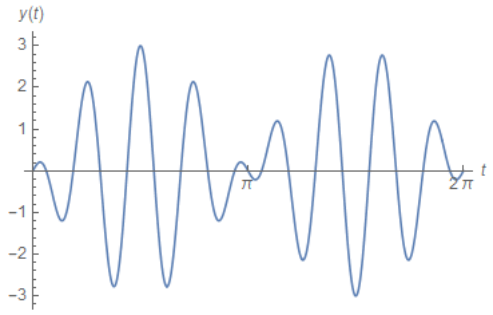
(a) A 1 kg mass is attached to a spring. When pushed, the following motion is observed (here $y(t)$ indicates displacement in meters):



(i) What is the spring constant k?

(ii) What is the (exact) initial velocity?

(b) A different mass-spring system is subjected to an oscillating external force, producing the following motion:



Write down a **possible** equation for $y(t)$ from the graph. You don't have to get it exactly right, and there is more than one possible answer.

$y(t) =$

Q4 (10 points)

(a) Solve the initial value problem.

$$y'' + \alpha^2 y = 1, \qquad y(0) = 0, \quad y'(0) = 0,$$

where $\alpha > 0$ is a constant. Your solution $y(t)$ will involve the symbol α. (Hint: this is an inhomogeneous equation)

(b) Find $\lim_{\alpha \to 0} y(t)$, where $y(t)$ is your solution from part (a).

Q5 (10 points)

A block of mass m is attached to a spring with spring constant $k = 1$ kg/s^2 and damping coefficient $\gamma = 4$ kg/s.

(a) For what range of values of m will the system exhibit oscillations with decaying amplitude?

(b) Interestingly, there is one particular value of m that will cause the system to oscillate with a larger quasifrequency ω than any other value of m. Find this value of m.