Name: _____ Student number: _____

Mathematics 207 J
University of Washington

February 23, 2022

MIDTERM 2

Here are the rules:

- This exam is closed book. No note sheets, calculators, or electronic devices are allowed.
- In order to receive credit, you must **show all of your work**; to obtain full credit, you must provide mathematical justifications. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Give numerical answers in exact form (for example $\ln(\frac{\pi}{3})$ or $5\sqrt{3}$ or $e^{2.5}$).
- If you need more room, use the backs of the pages and indicate that you have done so.
- This exam has 5 pages, plus a cover sheet. Please make sure that your exam is complete.

$$\cos(\alpha - \beta) = \cos\alpha\,\cos\beta + \sin\alpha\,\sin\beta$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\,\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\,\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\,\sin\frac{\alpha-\beta}{2}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\,\cos\frac{\alpha-\beta}{2}$$

Problem	Possible	Score
1	12	
2	10	
3	7	
4	14	
5	12	
Total	55	

Problem 1. (12 points)

(a) (9 points) Solve the following initial value problem.

$$y'' + 2y' + 5y = 5\cos t, \qquad y(0) = 1, \quad y'(0) = 0.$$

(b) (3 points) Identify the transient and the steady state components of your answer.

Problem 2. (10 points) Solve the following initial value problem:

$$y'' + 25y = \sin(5t), \qquad y(0) = 0, \quad y'(0) = 0.$$

Problem 3. (7 points) A spring is observed to stretch $\frac{1}{2}$ meter when a force of 3 newtons is applied to it. A viscous damper is observed to yield a resistance of 2 newtons when it is moved at a velocity of 1 meter/second.

A mass of 2 kg is hung from the spring and attached to the viscous damper. It is then pulled $\frac{1}{2}$ meter below its rest position and released with 0 initial velocity.

Write down the differential equation and initial conditions for $u(t)$, the position of the mass at time t relative to its rest position, where $u > 0$ means the mass is above the rest position. **Do not solve the equation**. **(And yes, this problem is really short.)**

Problem 4. (14 points) Consider the initial value problem

$$u'' + 2u' + \tfrac{5}{4}u = 0\,, \qquad u(0) = 2\,, \quad u'(0) = 1.$$

(a) (5 points) Solve the initial value problem.

$u(t) =$

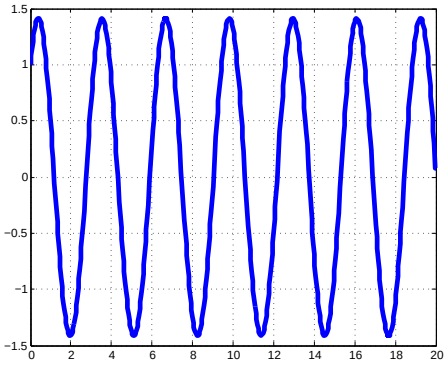
(b) (6 points) Express the answer in the form $u(t) = Ae^{\rho t}\cos(\omega t - \phi)$, where $A > 0$.

$A =$ $\rho =$ $\omega =$ $\phi =$

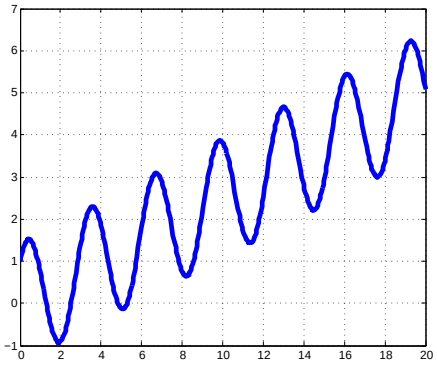
(c) (3 points) Find the first time $t > 0$ at which $u(t) = 0$.

$t =$

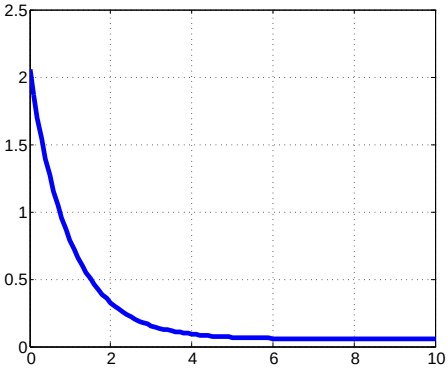
Problem 5. (12 points) Each of the 6 differential equations below has a solution that is plotted in one of the graphs. Match each of the differential equations to its solution. (Note: only 6 of the graphs will correspond to an equation.)



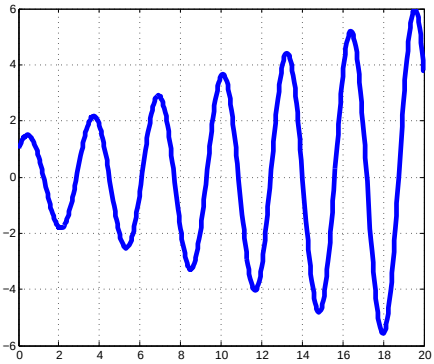
(a)



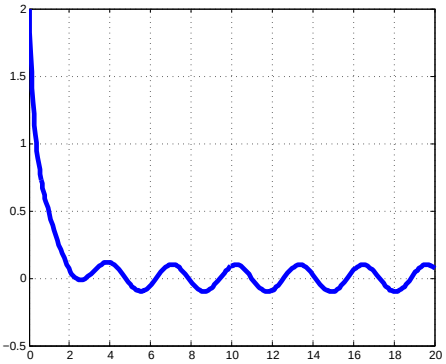
(b)



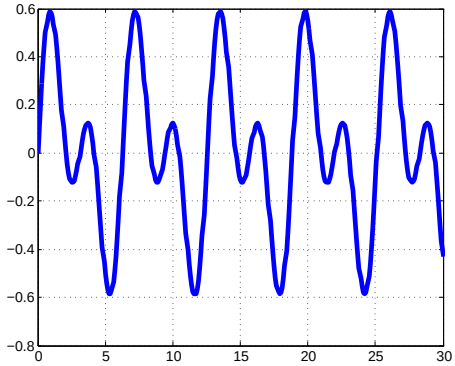
(c)



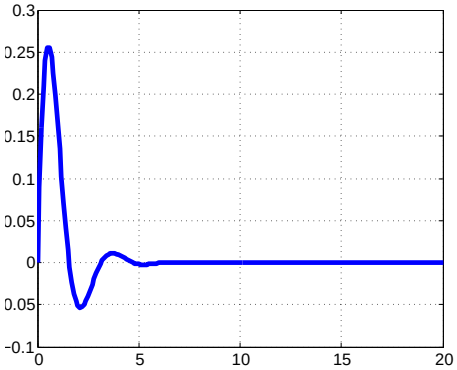
(d)



(e)



(f)



(g)

Differential Equation	Graph
$y'' + 4y = \sin(t)$	
$y'' + 4y = \cos(2t)$	
$y'' + 4y = 0$	
$y'' + 5y' + 4y = 0$	
$y'' + 5y' + 4y = \cos(2t)$	
$y'' + 2y' + 5y = 0$	

Submitted by Name: _____ *Student number:* _____
on February 23, 2022.