Math 126 Winter 2005 **Name:** ____________________

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Work Sheet #9

This worksheet is designed to help you review for the final. Discuss solutions with you group, continue working on the worksheet at home, bring questions to the following week's lectures.

1. Determine whether the series is convergent or divergent.
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{e^{2n+1}}$
   (b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3+(-1)^n}{3}\right)^n$
   (c) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{3n+1}\right)$
   (d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{\pi}{2} - \arctan n\right)$
2. For what values of x does the series $\sum_{n=1}^{\infty} (\cos x)^n$ converge?
3. Find the second degree Taylor polynomial centered at $a = 1$ for the function $f(x) = e^{x^2-1}$.
4. Let $f(x) = x^3 \cos(5x^2)$. Write down the Taylor series about $a = 0$ for the indefinite integral $\int f(x)\,dx$.
5. Consider the function $f(x) = \ln(3 + 2x^2)$.
   (a) Compute $f'(x)$ and find its Taylor series centered at zero.
   (b) Use part (a) to find the Taylor series centered at zero for $f(x)$. (**Hint:** What is $f(0)$?)
   (c) What is the radius radius of convergence of the series you found in part (b)?
   (d) We can approximate $f(x)$ by its 4th degree Taylor polynomial centered at 0. Find an interval around zero on which this approximation has error $< 10^{-6}$.
6. Suppose that the four vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$ in $\mathbf{R}^3$ are coplanar (i.e., that they all lie in the same plane). Show that $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) = \mathbf{0}$.
7. Find an equation for the plane through the origin that is perpendicular to the planes $5x - y + z = 1$ and $2x + 2y - 3z = 2$.
8. Find the parametric and symmetric (Cartesian) equations for a line that lies in the plane $2x - y + z = 4$ and intersects the line $\mathbf{r}(t) = \langle t, t, t\rangle$ at an angle of $\pi/2$.
9. Consider the ellipse $x^2 + \dfrac{y^2}{4} = 1$.
   (a) Write a parametrization for the closed path of a particle that starts at point $(-1, 0)$ and goes around the ellipse in the clockwise direction.
   (b) Find the area of the ellipse.

(c) Obtain an expression for the length of the ellipse. (Set up the integral, but do NOT compute it!)

10. Consider the space curve represented by the vector function $\mathbf{r}(t) = \langle \cos(t), \cos(t), \sqrt{2}\sin(t) \rangle$, where $0 \le t \le 2\pi$.

    (a) Compute $\mathbf{r}'(t)$.

    (b) Reparametrize the curve with respect to the arclength.

    (c) Let $P = (1/2,\ 1/2,\ \sqrt{3/2})$. Find the following.

    i. A parametrization of the tangent line for the curve at P.

    ii. The curvature of the curve at P.

    iii. An equation of the osculating plane for the curve at P.

    iv. An equation of the normal plane for the curve at P.

11. The position function of a spaceship is $\mathbf{r}(t) = \langle 3+t, 2+\ln t, 7+t^2 \rangle$ and the coordinates of the space station are $(7, 5, 14)$. The captain wants the spaceship to coast into the space station. When should the engines be turned off?

12. Find the vector function $\mathbf{r}(t)$ such that the acceleration is $\mathbf{a}(t) = \mathbf{i} - 12t^2\mathbf{j} + 2t\mathbf{k}$ and the initial position and velocity are given by $\mathbf{r}(0) = \mathbf{i} + \mathbf{k}$ and $\mathbf{v}(0) = 2\mathbf{j}$.

13. Consider the function $f(x, y) = e^{3x+5y-1}$.

    (a) Sketch the level sets of f, $f(x, y) = k$, for $k = e^{-1}$ and $k = 1$. What are the level sets if $k \le 0$?

    (b) Calculate the partial derivatives f_x and f_y.

    (c) Write an equation for the tangent plane to the graph of $f(x, y)$ at the point $(2, -1, 1)$.

    (d) Use the linear approximation for f at $(2, -1)$ to estimate the value $f(1.8, -0.9)$.

14. Integrate the function $f(x, y) = x+y$ over the region bounded by $x+y = 2$ and $y^2-2y-x = 0$.

15. Evaluate the following iterated integral:

$$\int_0^3 \int_0^{9-x^2} \frac{xe^{3y}}{9-y}\, dy\, dx$$