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Work Sheet #8

This worksheet is designed to help you review for the midterm. Discuss solutions with you group, continue working on the worksheet at home, bring questions to Thursday's quiz section.

1. Give an example of the following:

   (a) Three non-zero vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ such that $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$ but $\mathbf{b} \neq \mathbf{c}$.

   (b) A vector of length 5 that is orthogonal to $\mathbf{i} + 2\mathbf{j}$. How many such vectors are there in the xy-plane? How many such vectors are there in the entire three-dimensional space?

2. Suppose you are given non-zero vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ in three-dimensional space. Use dot- and cross-products and multiplication by scalars to give expressions for vectors satisfying the following geometric descriptions:

   (a) A vector of length 2 that is orthogonal to $\mathbf{a}$ and $\mathbf{b}$.

   (b) A vector with the same length as $\mathbf{b}$ and the same direction as $\mathbf{a}$.

   (c) A vector that is orthogonal to $\mathbf{a}$ and $\mathbf{b} \times \mathbf{c}$.

3. for each pair of objects, indicate whether they are parallel, perpendicular, or neither.

   (a) If $|\text{proj}_{\mathbf{a}}\mathbf{b}| = |\mathbf{b}|$, then vectors $\mathbf{a}$ and $\mathbf{b}$ are
   parallel **perpendicular** **neither**

   (b) If $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \mathbf{0}$, then vector $\mathbf{c}$ and the plane defined by vectors $\mathbf{a}$ and $\mathbf{b}$ are
   parallel **perpendicular** **neither**

   (c) The planes $x - 2y + 3z = 5$ and $2x - 4y + 6z = 5$.
   same **parallel** **perpendicular** **neither**

   (d) The planes $x - 2y + 3z = 5$ and $2x - 4y + 6z = 10$.
   same **parallel** **perpendicular** **neither**

   (e) The plane $x - 2y = 5$ and the x-axis (or the y or z-axis).
   parallel **perpendicular** **neither**

   (f) The plane $x - 2y = 5$ and the xy-plane (or the other coordinate planes).
   parallel **perpendicular** **neither**

4. (a) Find a vector function $\mathbf{r}(t)$ that represents the curve of intersection of the plane $z = 16$ and the surface $z = x^2 + 4y^2$.

   (b) Compute $\mathbf{r}'(t)$.

   (c) Write down but do not evaluate an integral for the length of the curve.

   (d) Find a vector equation for the tangent line to the curve at the point $(2, \sqrt{3}, 16)$.

   (e) Find the equation of a plane parallel to the x-axis and containing the points $A(0, 0, 3)$ and $B(0, 5, 0)$.

   (f) Find the point where the line of part (d) intersects the plane of part (e). (Do not expect "nice numbers".)

5. Find a parametrization $\mathbf{r}(t)$ of the curve that traces the circle $(x-5)^2+y^2 = 1$ in the clockwise direction starting at the point $(4, 0)$ (that is, $\mathbf{r}(0) = \langle 4, 0\rangle$).

6. (a) A particle moves along the helix so that its position vector at time t is $\mathbf{r}(t) = \langle 4\cos(t),\ 4\sin(t),\ 3t\rangle$. Compute the curvature $\kappa(t)$ of the helix.

   (b) Reparametrize the helix by arclength.

   (c) Assume now that another particle moves along the same path, but its position at time τ is described by $\mathbf{q}(\tau) = \langle 4\cos(\tau^5),\ 4\sin(\tau^5),\ 3\tau^5\rangle$. Compute the length of $\mathbf{q}'(\tau) \times \mathbf{q}''(\tau)$. **Hint:** do not compute $q''(\tau)$. Instead use the information you computed in part (a). (What characteristic of the curve does not depend on its parametrization?)