

**Worksheet 6 Math 126**  
(best if discussed in small groups)

This worksheet gives you some practice with vector functions, in particular velocity vector functions.

1. If the position of a particle as a function of time  $t$  is given in polar coordinates by  $(r(t), \theta(t))$ , then find the velocity vector function in polar coordinates. If we use the notation  $(s(t), \alpha(t))$  for the velocity vector, then  $s(t)$  is the magnitude of the velocity vector and is called the speed (note that  $r'(t)$  is not the speed) and the angle  $\alpha$  of the velocity vector gives the direction of travel. Hint: One way to do this is to convert the position to Cartesian coordinates, use the chain rule to find the velocity, then convert the result back to polar coordinates. A more geometric way to find the speed would be to observe that the rate of change in distance to the origin is  $r'(t)$  and the rate of change in distance perpendicular to the radius vector is  $r\theta'(t)$ . Use these two pieces of information to find the speed. Can you find the direction  $\alpha$  geometrically? Hint:

$$\alpha(t) = \theta(t) + \tan^{-1}\left(\frac{r'(t)}{r(t)\theta'(t)}\right).$$

2. A plane flies at a speed of 180 km/hr in still air. The pilot heads due north according to the plane's compass. After 30 minutes of flight time, the pilot notices that, due to the wind, the plane has actually traveled 80 km at an angle  $5^\circ$  east of north.
  - (a) What is the wind velocity? Hint: think of the airplane as flying within a body of air. The vector describing the wind speed and direction gives the speed and directions of the body of air. The airplane however flies 180km/hr *relative to the body of air* (think of a new coordinate system that moves with the wind). In this sense, the velocity vector due to the airplane's engine can be added to the velocity vector due to the wind. Velocities are easier to add in Cartesian coordinates.
  - (b) If the pilot were given the wind velocity (which you found in part (a)) at the start, in what direction should the pilot head to fly 90 miles due north and how long will it take him to get there?