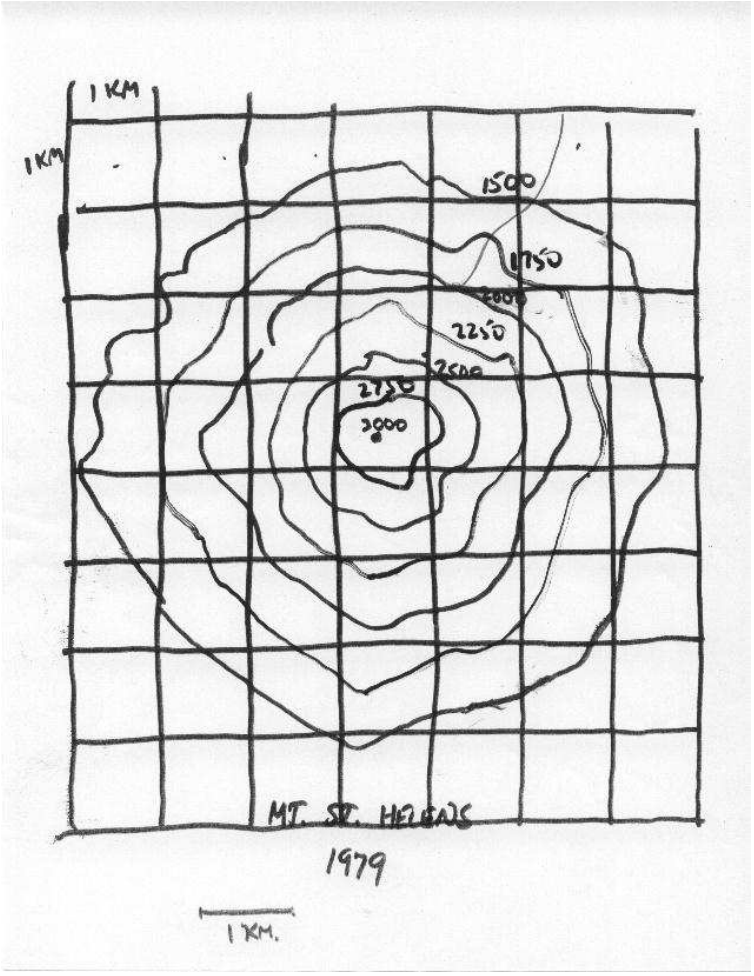
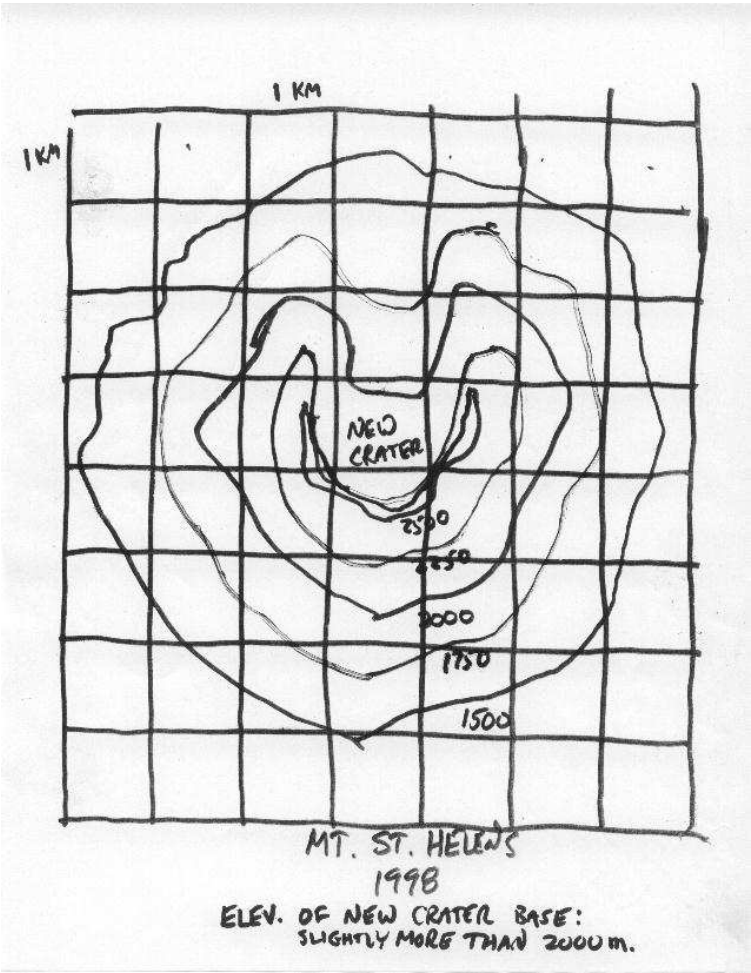
Worksheet 9 Math 126
(best if discussed in small groups)

This worksheet gives an example of estimating a double integral by Riemann sums, and shows the benefit of using linearity.



1. Figure 1 is a (somewhat crude) topographical map of Mt. St. Helens in 1979, prior to the eruptions. The curves are level lines. The numbers near each level line give the altitude or elevation of the mountain above the level line. If $h(x, y)$ denotes the altitude above the point (x, y), then why does $\int\int_R h(x, y)dxdy$ represent the volume of the mountain above the region R? (Hint: how is this integral defined?)



2. Figure 2 is a topographical map in 1998, after the eruptions. Where on the map is the mountain unchanged and where is it changed? Where is the new mountain very steep? The base of the crater has elevation slightly more than 2000m.

3. If $k(x, y)$ denotes the altitude of the mountain above the point (x, y) then the loss of volume is given by:

$$L = \int\int_R h(x, y)dxdy - \int\int_R k(x, y)dxdy. \tag{1}$$

We can estimate each integral by calculating a Riemann sum over the grid of squares, and then estimate the loss of volume by subtracting the two sums. It is easier to first note that

$$L = \int\int_R \big(h(x, y) - k(x, y)\big)dxdy, \tag{2}$$

because for most squares the altitude is unchanged and so $h(x, y) - k(x, y) = 0$ on those squares. Each square in the grid is one square kilometer. The altitudes are given in meters. For each square in the grid, estimate the change in altitude. There is

no "right" answer from the picture, just find a number for each square that you think best estimates the (average) loss in altitude. Use these numbers to form a Riemann sum for (2). The USGS estimates that the loss of volume was approximately 4 cubic kilometers. They use smaller squares in their grid, which would be too time consuming in this quiz section.

3. A creek is drawn in the northwest corner of Figure 1. Why does it appear that the creek flows perpendicular to the level curves? (this is a way of recognizing creeks on a heavily detailed topographical map). When the ice melts in the new crater, which way does it flow out?