Worksheet 2 Math 126

1. Use the Taylor Series for the exponential function to do the following.
   (a) Find the Taylor Series based at zero for $f(x) = e^{x^3-1} = e^{x^3}/e$. What is the value of $\frac{d^{15}f}{dx^{15}}$ at $x = 0$?
   (b) Find the Taylor Series based at zero for $g(x) = e^{(x/a)^2-b}$.
   (c) Find the Taylor Series based at zero for $h(x) = xe^{(x/a)^2-b}$. Hint: How is h related to g in part (b)?
   (d) Find the Taylor Series based at 1 for $f(x) = xe^x$.

   Here is an application of Taylor polynomials:

2. An electric dipole consists of two electric charges of equal magnitude and opposite signs. If the charges are q and $-q$ and are located at a distance d from each other, then the electric field E at the point P in the figure

   

   is given by

$$E = \frac{q}{D^2} - \frac{q}{(D+d)^2} = \frac{q}{D^2}\left[1 - \frac{1}{(1+\frac{d}{D})^2}\right].$$

   Treating d/D as a variable (set $x = d/D$) write down the Taylor expansion (Taylor Series) for the function inside the square brackets. Multiply the Taylor expansion by q/D^2 to obtain an expansion for E. Using the expansion you obtained, show that E is approximately $2qd/(D^3)$ when P is far away from the dipole.

   Comment: The electric field decays much more rapidly than the field generated by one of the charges because the fields generated by each particle partially cancel one another. The approximation tells us the rate of decay.