Math 126 - Winter 2007
Exam 2
February 22, 2007

Name: __________________________________________
Section: __________________________________________
Student ID Number: __________________________________

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- You are allowed to use a scientific calculator (no graphing calculators) and one hand-written 8.5 by 11 inch page of notes. Put your name on your sheet of notes and turn it in with the exam.
- Check that your exam contains all the problems listed above.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Use your time wisely.

GOOD LUCK!
1. (10 points) A particle’s position in two dimensions is given by the parametric equations

\[
\begin{align*}
x &= \ln(t) \\
y &= 2t^3 - 21t^2 + 60t,
\end{align*}
\]

where \( t > 0 \).

(a) (5 points) Find all points \((x, y)\) where the parametric curve has a horizontal tangent.

(b) (5 points) Find the velocity and acceleration vectors at the point \((0, 41)\).
2. (8 points) Let \( r \) be an unknown constant.

(a) (5 points) Find the equation of the plane through the points \((0, 2, 0), (1, 0, 0), \) and \((0, 1, r)\).
Write your answer in the form \( ax + by + cz + d = 0 \).
(Your answer will be the equation of a plane and will involve the constant \( r \).)

(b) (3 points) If \((3, 2, 1)\) is also a point on the plane, what is the value of \( r \)?
3. (12 points) Let \( \mathbf{r}(t) = \langle 3 \cos(t), 5 \sin(t), 4 \cos(t) \rangle \).

(a) (4 points) Show that the unit tangent vector \( \mathbf{T}(t) \) is orthogonal to \( \mathbf{r}''(t) \) for all \( t \).

(b) (4 points) Find the parametric equations for the tangent line to the curve at \( t = \frac{\pi}{2} \).

(c) (4 points) Find the curvature \( \kappa(t) \).
4. (10 points) The acceleration for a particle is given by the vector function \( a(t) = \langle t, 0, -1 \rangle \) for \( t \geq 0 \). The initial position of the particle \( r(0) = \langle 1, 0, 1 \rangle \) and the initial velocity is \( v(0) = \langle -2, 1, 0 \rangle \).

(a) (4 points) Find the position vector \( r(t) \).

(b) (2 points) Will the particle ever be located at the origin? If not, explain why. If so, give all times when the particle is at the origin.

(c) (4 points) Find all times when the tangential component of the acceleration vector is zero.
5. (10 points) Let \( f(x, y) = x^2 y + x \ln(x + y) \)

(a) (2 points) Find and sketch the domain of the function.
   Clearly indicate everything that is included in the domain in your sketch.

(b) (4 points) Compute the partial derivatives of \( f_x(x, y) \) and \( f_y(x, y) \).

(c) (4 points) Compute the second partial derivatives \( f_{xy}(x, y) \) and \( f_{yy}(x, y) \).