• Turn off and put away cell phones, graphing calculators, books, and notebooks.

• You may use one 8$\frac{1}{2}$ × 11 sheet of handwritten notes and a non-graphing calculator. Do not share notes or calculators.

• In order to receive credit, you must show your work and explain your reasoning (unless the problem instructions say otherwise).

• Give exact answers. You do not need to simplify answers algebraically.

• Place a box around YOUR FINAL ANSWER to each question.

• Raise your hand if you have a question or need more paper.

Please do not open the test until everyone has a copy and the start of the test is announced.
1. (4 points) The contour maps for the functions $f$ and $g$ appear below. For each map, the level curves are drawn for evenly spaced values of the function. *No reasoning need be shown for this problem.*

The contour map of $f(x, y) = \ln(x^2 + y^2)$ is __________.

The contour map of $g(x, y) = x^2 + y^2$ is __________.

*NOTE FOR ON-LINE COPY OF TEST:*
The hard copy of the test has three contour maps, all consisting of concentric circles. Map A has evenly spaced circles. Map B has circles that are more closely spaced far from the origin. Map C has circles that are more closely spaced near the origin.
2. (12 points) Consider the curve described by \( \mathbf{r}(t) = \langle t + 3, e^t, t^2 - 1 \rangle \).

(a) Find the curvature at the point corresponding to \( t = 0 \).

(b) Find the equation of the osculating plane at the point corresponding to \( t = 0 \).
3. (12 points) A curve is given in polar coordinates by the equation $r = \theta$. Find the Cartesian equation of the tangent line to the curve at the point on the curve corresponding to $\theta = \pi/2$. 

4. (12 points) For the function \( f(x, y) = x^2 \sin(\pi y) \),

(a) Compute \( f_x(x, y) \), \( f_y(x, y) \), and \( f_{xy}(x, y) \).

(b) Find the equation of the tangent plane to the graph of \( f(x, y) \) at the point where \( (x, y) = (3, 1) \).

(c) Find the equations of the line through \( (3, 1, f(3, 1)) \) and perpendicular to the tangent plane in part (b).
5. (12 points) (a) Find parametric equations for the ellipse whose cartesian equation is 
\[ 4x^2 + 9(y + 1)^2 = 9 \] 
so that at \( t = 0 \) your equations have \((x(0), y(0)) = (-3/2, -1).\)

(b) Does your answer from part (a) describe motion going

\textit{CLOCKWISE} or \textit{COUNTERCLOCKWISE}?

Circle one of the words above; no explanation required.
6. (8 points) Suppose that the only force $\mathbf{F}$ acting on an object is parallel to the position function $\mathbf{r}(t)$ of the object. Prove that the angular momentum $\mathbf{L}(t) = m\mathbf{r}(t) \times \mathbf{v}(t)$ is constant.

(Use Newton’s second law, $\mathbf{F} = ma$. Physicists call a force that is parallel to the position vector a “central force.”)