1. **[8 points]** Write the equation of the plane tangent to $z = x^2y - x$ at the point $(2, 3, 10)$.

$\frac{\partial z}{\partial x} = 2xy - 1$, so at $(2,3,10)$, $\frac{\partial z}{\partial x} = 11$

$\frac{\partial z}{\partial y} = x^2$, so at $(2,3,10)$, $\frac{\partial z}{\partial y} = 4$

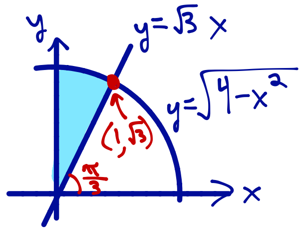
So the tangent plane is $\boxed{z = 10 + 11(x-2) + 4(y-3)}$

or $\boxed{z = 11x + 4y - 24}$

2. **[8 points]** An integral on the first page? Weird. Compute $\int_0^1 \int_{\sqrt{3}x}^{\sqrt{4-x^2}} x\sqrt{x^2+y^2}\,dy\,dx$.

Hm, sounds polar to me.

Let's draw a picture:



So this is

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_0^2 (r^3\cos\theta)\,dr\,d\theta = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(\frac{1}{4}\cos\theta\, r^4\right)\Big]_0^2 d\theta = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 4\cos\theta\,d\theta$$

$$= 4\sin\theta\Big]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = \boxed{4 - 2\sqrt{3}}$$

3. **[2 points each]** Here are six multivariable functions and their names:

(Andrea) $f(x,y) = x + y$ (Dorian) $f(x,y) = x - y$

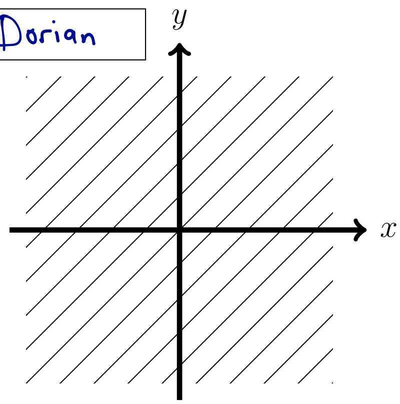
(Barry) $f(x,y) = x^2 + y^2$ (Erin) $f(x,y) = x^2 - 2xy + y^2$

(Chantal) $f(x,y) = \sqrt{x^2 + y^2}$ (Fernand) $f(x,y) = \sqrt[4]{x^2 + y^2}$

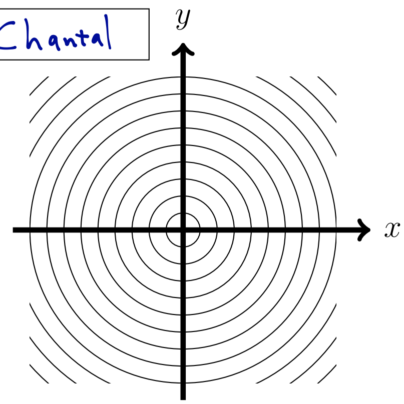
Write the name of each function in the box next to its level curves below.

You do not need to show any work for this problem.

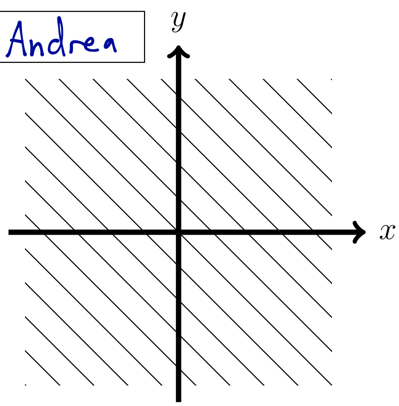
Dorian



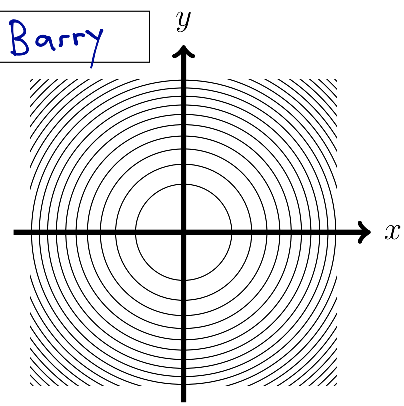
Chantal



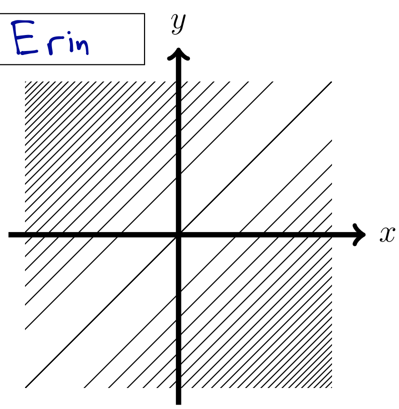
Andrea



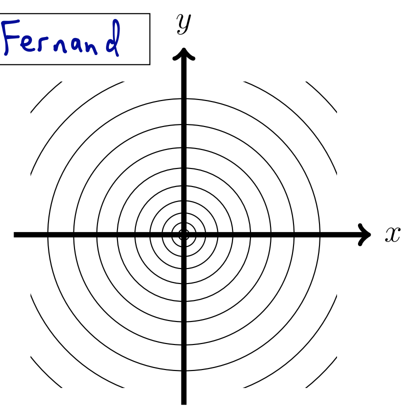
Barry



Erin



Fernand



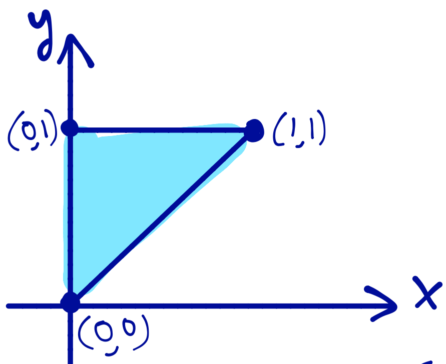
(The names are because last time I just used letters, and it's super hard to read some people's handwriting from just one letter.)

This problem is sponsored by the triangle T with vertices $(0,0)$, $(0,1)$, and $(1,1)$.

4. **[12 points]** Let $f(x,y) = x^2 + 2y^2 - xy - 2x - y$.

   Find the absolute minimum and maximum values of f on the domain T.

   (Just to be clear: T includes both the interior and the boundary of the triangle.)



Critical points inside:

$f_x(x,y) = 2x - y - 2 = 0 \to y = 2x - 2$

$f_y(x,y) = 4y - x - 1 = 0 \to 4(2x-2) - x - 1 = 0$

$7x - 9 = 0$

$x = \frac{9}{7}$, not in domain

So, none.

Sides

Left edge: $x = 0$

$f(0,y) = 2y^2 - y$

$f' = 4y - 1 = 0$, so $y = \frac{1}{4}$: check $(0, \frac{1}{4})$

Top edge: $y = 1$

$f(x,1) = x^2 - 3x + 1$

$f' = 2x - 3 = 0 \to x = \frac{3}{2}$, not in domain

Right edge: $y = x$

$f(x,x) = 2x^2 - 3x$

$f' = 4x - 3 = 0 \to x = \frac{3}{4}$: check $(\frac{3}{4}, \frac{3}{4})$

Also check vertices: $(0,0)$, $(0,1)$, $(1,1)$

Points to check:

$f(0,0) = 0$

$f(0,1) = \boxed{1}$ ← max

$f(1,1) = -1$

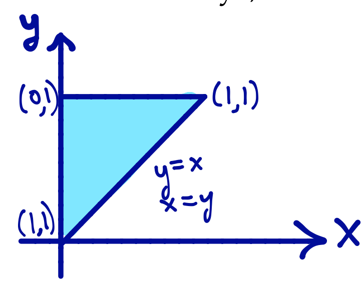
$f(0, \frac{1}{4}) = \frac{-1}{8}$

$f(\frac{3}{4}, \frac{3}{4}) = \boxed{\frac{-9}{8}}$ ← min

This problem is also sponsored by the triangle T with vertices $(0,0)$, $(0,1)$, and $(1,1)$.

5. Let $\mathcal{S}$ be the solid bounded above by $z = e^{y^2}$ and below by T in the xy-plane.

   (a) **[5 points]** Set up a double integral for the volume of $\mathcal{S}$ in two different ways, once using $dx\,dy$ and once using $dy\,dx$. (Don't evaluate it yet.)



$$\boxed{\int_0^1 \int_0^y e^{y^2}\,dx\,dy}$$

$$\boxed{\int_0^1 \int_x^1 e^{y^2}\,dy\,dx}$$

   (b) **[5 points]** Okay, now find the volume of $\mathcal{S}$. Use whichever setup you prefer.

$$\int_0^1 \int_0^y e^{y^2}\,dx\,dy = \int_0^1 \left(xe^{y^2}\right)\Big]_0^y dy = \int_0^1 ye^{y^2} = \frac{1}{2}\int_0^1 e^u\,du = \boxed{\frac{1}{2}(e-1)}$$

$u = y^2$

$du = 2y\,dy$

6. (a) **[7 points]** Find all the critical points (in $\mathbb{R}^2$) of $f(x, y) = x\sin(y) + y^2$.

(There are a lot of them! You should list them all somehow, but I don't really care about the format of your answer.)

$f_x(x,y) = \sin(y) = 0 \rightarrow y = \ldots -\pi, 0, \pi, 2\pi, \ldots$

$f_y(x,y) = x\cos(y) + 2y$

if $y = \ldots -\pi, \pi, 3\pi, \ldots$

then $\cos y = -1$

$x = 2y$

so $\ldots (-2\pi, -\pi), (2\pi, \pi), (6\pi, 3\pi), \ldots$

or...

if $y = \ldots 0, 2\pi, 4\pi, \ldots$

then $\cos y = 1$

$x = -2y$

so $\ldots (4\pi, -2\pi), (0,0), (-4\pi, 2\pi), \ldots$

In general: $\boxed{\ldots (-6\pi, -3\pi), (4\pi, -2\pi), (-2\pi, -\pi), (0,0), (2\pi, \pi), (-4\pi, 2\pi), \ldots}$

or if you're fancy: $\boxed{(-4k\pi, 2k\pi) \; \& \; (2(2k+1)\pi, (2k+1)\pi)}$

for integers k

(b) **[3 points]** Classify your critical points from part (a) as local maxima, local minima, or saddle points.

$f_{xx}(x,y) = 0$

$f_{xy}(x,y) = \cos y = \pm 1$ at each crit. point.

So $D(a,b) = 0 \cdot (\text{something}) - (\pm 1)^2 = -1$

so they're $\boxed{\text{all saddle points}}$