Math 126 A and B Midterm 2 Spring 2025

Print Your Full Name: Solutions

Signature:

Student ID Number:

Quiz Section:

Instructor's Name:

TA's Name:

Please read these instructions!

1. Your exam contains 8 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. You are allowed a single, double-sided 8.5"x11" handwritten notesheet; the TI-30XIIS calculator; a writing utensil; and an eraser to use on the exam.
3. The exam is worth 50 points. Point values for problems vary and these are clearly indicated. You have 80 minutes for this exam.
4. Unless otherwise stated, make sure to SHOW YOUR WORK CLEARLY. Credit is awarded to work which is clearly shown and legible. Full credit may not be awarded if work is unclear or illegible.
5. For problems that aren't sketches, place a $\boxed{\text{box around your final answer}}$ to each question.
6. If you need extra space, use the last two pages of the exam. Clearly indicate on that there is more work located on the last pages, and indicate on those pages the related problem number.
7. Unless otherwise instructed, always give your answers in exact form. For example, 3π, $\sqrt{2}$, and $\ln(2)$ are in exact form; the corresponding approximations 9.424778, 1.4142, and 0.693147 are NOT in exact form.
8. Credit is awarded for correct use of techniques or methods discussed in class thus far. Partial credit may be awarded as earned. No credit is awarded for use of methods that are learned later in the course or from other courses.

Problem	Total Points
1	10
2	10
3	10
4	10
5	10
Total	50

1. (10 points) Use the linearization of the function $f(x, y) = 1 - xy\cos(\pi y)$ at $(1, 1)$ to approximate the value of $f(1.02, 0.97)$. All expressions involving trigonometric function values on the unit circle must be fully simplified to exact form.

$f_x = -y\cos(\pi y)$ +2

$f_y = -xy(-\sin(\pi y)\pi) + \cos(\pi y)\cdot(-x)$ +2

$f(1,1) = 1 - \cos(\pi)$
$= 2$ +1

$f_x(1,1) = -\cos(\pi) = 1$
$f_y(1,1) = \pi\sin(\pi) - \cos(\pi) = 1$ +1

$L(x,y) = 1(x-1) + 1(y-1) + 2$ +2

$\Rightarrow f(1.02, 0.97) \approx L(1.02, 0.97)$
$= (1.02 - 1) + (0.97 - 1) + 2$
$= 0.02 - .03 + 2 = \boxed{1.99}$ +2

2. (10 points) Find the (x, y) points corresponding to the local maximum value, local minimum value, and saddle points of the function $f(x, y) = x^4 + 4x^2(y-2) + 8(y-1)^2$.

$f_x = 4x^3 + 8x(y-2) = 0$ $\rightarrow$ $4x\left(x^2 + 2(y-2)\right) = 0 \rightarrow x = 0$ or

$f_y = 4x^2 + 16(y-1) = 0$ +1 $x^2 + 2y - 4 = 0$

$x^2 = 4 - 2y$

$x = 0$: $16(y-1) = 0 \rightarrow y = 1$ crit. pt $(0,1)$ +1

$x^2 = 4 - 2y$: $4(4-2y) + 16(y-1) = 0$

$\cancel{16} - 8y + 16y - \cancel{16} = 0$

$8y = 0 \rightarrow y = 0$

$x^2 = 4$

$x = \pm 2$ crit. pt. $(\pm 2, 0)$ +2

$f_{xx} = 12x^2 + 8(y-2)$ $f_{yy} = 16$ $f_{xy} = 8x$

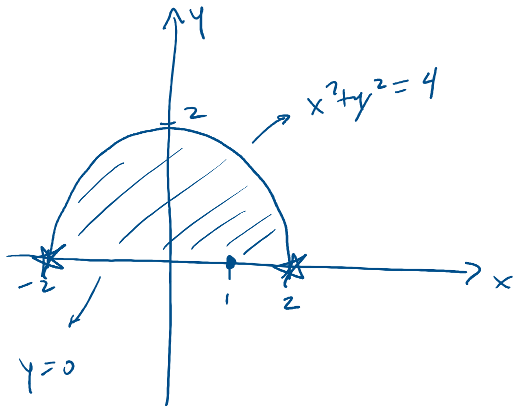
$D = \left(12x^2 + 8(y-2)\right) \cdot 16 - (8x)^2$

$D(0,1) < 0 \Rightarrow \boxed{\text{saddle point at } (0,0)}$ +2

$D(2,0) = 12(2)^2 \cdot 16 - 16^2 > 0$, $f_{yy} > 0 \Rightarrow \boxed{\text{local min value at } (2,0).}$ +2

$D(-2,0) = D(2,0) > 0$, $f_{yy} > 0 \Rightarrow \boxed{\text{local min value at } (-2,0).}$ +2

3. (10 points) Find the absolute maximum and minimum values of the function $f(x, y) = x^2 + y^2 - 2x + 2$ on the domain $D = \{(x, y) : x^2 + y^2 \leq 4, y \geq 0\}$.



$f_x = 2x - 2 = 0 \to x = 1$

$f_y = 2y = 0 \to y = 0$ (+1)

No crit. pts inside domain.

$\underline{y=0}$: $f(x,0) = x^2 - 2x + 2 = g(x)$ $\quad -2 \leq x \leq 2$

$g' = 2x - 2 = 0 \to x = 1$ $\quad (1, 0)$ (+2)

☆ End points: $\underbrace{(-2, 0) \quad (2, 0)}_{+2}$

$\underline{x^2+y^2=4}$: $4 - 2x + 2 = g(x)$

$6 - 2x = g(x)$ $\quad -2 \leq x \leq 2$

$g' = -2$ No additional critical numbers. (+1)

Check: $f(1, 0) = 1 + 0 - 2 + 2 = \boxed{1}$ ← abs min value (+1)

$f(-2, 0) = 4 + 0 - 2(-2) + 2 = \boxed{10}$ ← abs max value (+1)

$f(2, 0) = 4 + 0 - 2(2) + 2 = 2$

(+2)

(+1) only, if testing other points

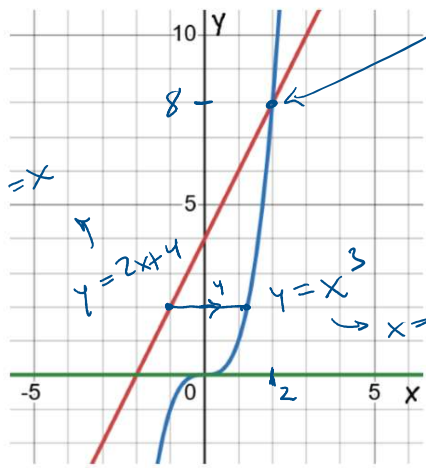
4. Consider the region D in the xy-plane bounded by the curves $y = 0, y = 2x + 4$ and $y = x^3$. (See the figure below.) **Set up (but do not evaluate)** an iterated integral, or sum of iterated integrals, in the indicated orders below equal to the integral $\iint_D \cos(x)y^2 \, dA$.

(a) (5 points) In the order of $dx\, dy$

$2x+4 = x^3 \rightarrow x = 2$ (intersection in 1st quadrant)

$0 = x^3 - 2x + 4$

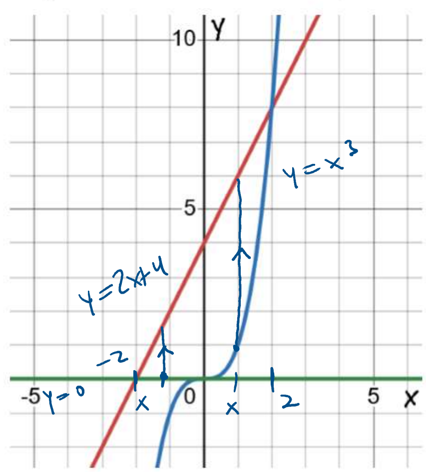
$\frac{y-4}{2} = x$



$$\int_0^8 \int_{\frac{y-4}{2}}^{\sqrt[3]{y}} \cos(x) y^2 \, dx\, dy$$

(+2) (+2) (+1)

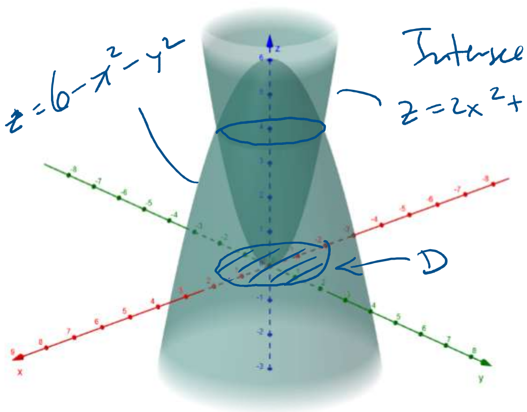
(b) (5 points) In the order of $dy\, dx$



$$\int_{-2}^{0} \int_{0}^{2x+4} \cos(x) y^2 \, dy\, dx + \int_0^2 \int_{x^3}^{2x+4} \cos(x) y^2 \, dy\, dx$$

(+1) (+1) (+1) (+1) (+1)

5. (10 points) Use polar coordinates to find the volume of the solid bounded by the paraboloids $z = 6 - x^2 - y^2$ and $z = 2x^2 + 2y^2$. (See the figure below.)



Intersection: $6 - x^2 - y^2 = 2x^2 + 2y^2$

$6 = 3x^2 + 3y^2$

$2 = x^2 + y^2$ +2

circle radius $\sqrt{2}$

$$\text{Volume} = \iint_D \left(6 - x^2 - y^2 - (2x^2 + 2y^2)\right) dA$$

$$= \iint_D (6 - 3x^2 - 3y^2)\, dA$$

$6 - 3\underbrace{(x^2 + y^2)}_{= r^2}$

$$= \boxed{\int_0^{2\pi} \int_0^{\sqrt{2}}} (6 - 3r^2)\, r\, dr\, d\theta$$

+2 +2 +2

Credit given for integrand $3r^2 - 6$, if absolute value taken at end.

$$= \int_0^{2\pi} \int_0^{\sqrt{2}} (6r - 3r^3)\, dr\, d\theta$$

$$= \int_0^{2\pi} \left[3r^2 - \frac{3r^4}{4}\right]_0^{\sqrt{2}} d\theta$$

$$= \int_0^{2\pi} 6 - \frac{3 \cdot \cancel{4}}{\cancel{4}} - 0 \, d\theta$$

$$= \int_0^{2\pi} 3\, d\theta = \boxed{6\pi}$$ +2

Extra scratch paper.

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