1. (14 pts) **Part (a) and (b) below are not related.**

(a) An object's position at time t (where $t > 0$) is $\mathbf{r}(t) = \langle 5t, t^{(3/2)}, \frac{1}{\sqrt{3}}\ln(t)\rangle$.

i. Find the time(s) at which the acceleration vector is orthogonal to its velocity vector.

$$\vec{r}\,'(t) = \langle 5, \tfrac{3}{2}t^{1/2}, \tfrac{1}{\sqrt{3}}\tfrac{1}{t}\rangle$$

$$\vec{r}\,''(t) = \langle 0, \tfrac{3}{4}t^{-1/2}, -\tfrac{1}{\sqrt{3}}\tfrac{1}{t^2}\rangle$$

$$\vec{r}\,' \cdot \vec{r}\,'' = \frac{9}{8} - \frac{1}{3}\frac{1}{t^3} \overset{?}{=} 0$$

$$\Rightarrow \frac{9}{8} = \frac{1}{3}\frac{1}{t^3}$$

$$\Rightarrow t^3 = \frac{8}{27}$$

$$\Rightarrow \boxed{t = 2/3}$$

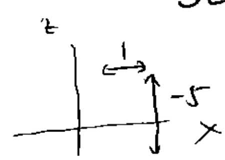
ii. At the time(s) you found in previous part, the vector $\mathbf{r}''(t)$ is parallel to at least one of the vectors in the TNB-Frame at that same time. Which one?

No work needed, just circle your answer: **T** **N** **B.**

(b) Consider the curve of intersection of the surface $e^{3z} = x^2z + \ln(y) + 5x - 10$ and the *fixed* plane $y = 1$. Find 3D parametric equations for the tangent **line** to this curve at the point $(2, 1, 0)$. (Hint: Start by using implicit differentiation to find a partial derivative).

$$3e^{3z}\frac{\partial z}{\partial x} = 2xz + x^2\frac{\partial z}{\partial x} + 5 \Rightarrow \frac{\partial z}{\partial x} = \frac{2xz+5}{3e^{3z} - x^2}$$

$$\left.\frac{\partial z}{\partial x}\right|_{(2,1,0)} = \frac{2(2)(0)+5}{3e^0 - (2)^2} = \frac{5}{3-4} = -5$$



slope in x-direction $= -5$

vector in x-direction $= \langle 1, 0, -5\rangle$

$$\boxed{\begin{aligned} x &= 2 + t \\ y &= 1 \\ z &= 0 - 5t \end{aligned}}$$

2. (10 pts) The total surface area of a solid cone with radius r and height h is given by

$$A(r,h) = \pi r^2 + \pi r\sqrt{r^2+h^2}.$$

(a) Find the equation for the tangent plane to $A(r,h)$ when $r = 3$ inches and $h = 4$ inches.

$A(3,4) = 9\pi + \pi(3)(5) = 24\pi$ $\qquad \sqrt{3^2+4^2} = 5$

$A_r = 2\pi r + \pi\sqrt{r^2+h^2} + \dfrac{2\pi r^2}{2\sqrt{r^2+h^2}} \Rightarrow A_r(3,4) = 6\pi + 5\pi + \dfrac{9\pi}{5} = \dfrac{64}{5}\pi$

$A_h = \dfrac{2\pi r h}{2\sqrt{r^2+h^2}} \Rightarrow A_h(3,4) = \dfrac{12\pi}{5}$

$$\boxed{A - 24\pi = \frac{64}{5}\pi(r-3) + \frac{12\pi}{5}(h-4)}$$

(b) Use the total differential to approximate the *change* in surface area if r is increased from 3 to 3.2 inches and h is increased from 4 to 4.1 inches.

$dr = 0.2,\ dh = 0.1$

$dA = \frac{64}{5}\pi\, dr + \frac{12}{5}\pi\, dh = \frac{1}{5}\pi(64(0.2) + 12(0.1)) = \frac{1}{5}\pi(12.8 + 1.2) = \frac{14}{5}\pi$

$\boxed{dA = \frac{14}{5}\pi \text{ in}^2} = 2.8\pi \approx 8.7964\ldots$

ASIDE: ACTUAL VALUE $= A(3.2, 4.1) - A(3,4) \approx 9.06 \text{ in}^2$

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3. (~~10~~ pts) **The two parts below are not related.**

(a) Let $f(x, y) = yx^2 + x^3 - 4y$. Find and classify all the critical points of $f(x, y)$. Show your work in using the second derivative test.

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$f_x = 2yx + 3x^2 \stackrel{?}{=} 0$

$f_y = x^2 - 4 \stackrel{?}{=} 0 \Rightarrow x = \pm 2$

$x = 2 \Rightarrow 2y(2) + 3(2)^2 \stackrel{?}{=} 0 \Rightarrow 4y = -12 \quad y = -3 \qquad (2, -3)$

$x = -2 \Rightarrow 2y(-2) + 3(-2)^2 \stackrel{?}{=} 0 \Rightarrow -4y = -12 \quad y = 3 \qquad (-2, 3)$

$f_{xx} = 2y + 6x$

$f_{yy} = 0$

$f_{xy} = 2x$

$D = f_{xx} f_{yy} - f_{xy}^2 = 0 - (2x)^2$

FOR $(2,-3)$ AND $(-2,3)$ $\quad D = -16 < 0$

BOTH SADDLE POINTS

(b) Suppose that $f(x, y)$ is a continuous function and that $\iint_D f(x, y)dA = \int_1^3 \int_7^{16-x^2} f(x, y)dydx$. Sketch the region D and reverse the order of integration (i.e. rewrite the integral and give the bounds for the order $dxdy$).

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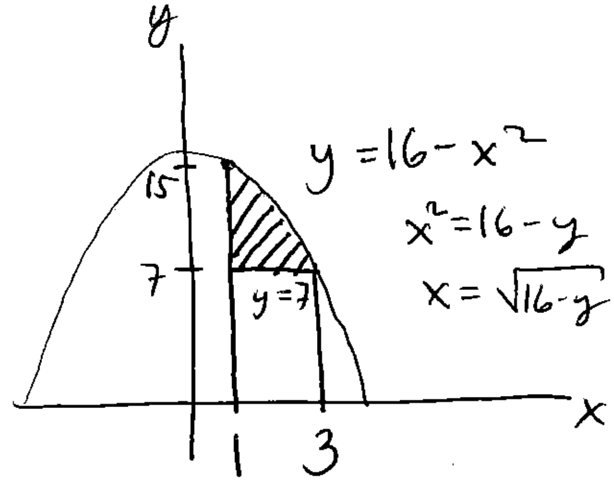
$1 \leq x \leq 3$

$7 \leq y \leq 16 - x^2$

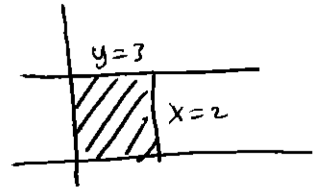
$$\int_7^{15} \left(\int_1^{\sqrt{16-y}} f(x,y)\, dx \right) dy$$



4. (14 pts) **The two problems below are not related.**

(a) Find the volume of the solid bounded by $z = 9 - x^3$, $z = 1$, $x = 0$, $y = 0$, and $y = 3$.

$$\iint_D 9 - x^3 \, dA - \iint_D 1 \, dA = \iint_D 8 - x^3 \, dA$$



$$\int_0^3 \int_0^2 8 - x^3 \, dx \, dy$$

$$= \int_0^3 8x - \tfrac{1}{4}x^4 \Big|_0^2 \, dy$$

$$= \int_0^3 16 - 4 \, dy$$

$$= 12y \Big|_0^3 = \boxed{36}$$

INTERSECTION OF

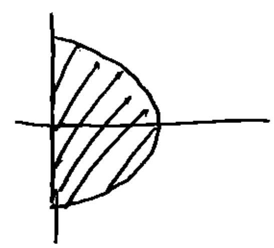
$z = 1$ & $z = 9 - x^3$

IS $1 \overset{?}{=} 9 - x^3$

$x^3 = 8 \Rightarrow x = 2$

(b) Evaluate $\displaystyle\iint_D x e^{(x^2+y^2)^{3/2}} dA$, where D is the region bounded by the semicircle $x = \sqrt{4 - y^2}$ and the y-axis. (Hint: Polar would be a good choice.)

$$\int_{-\frac{\pi}{2}}^{\pi/2} \int_0^2 r\cos\theta \, e^{r^3} \, r \, dr \, d\theta$$



$$\int_{-\pi/2}^{\pi/2} \cos\theta \int_0^2 r^2 e^{r^3} dr \, d\theta$$

$u = r^3$

$du = 3r^2 dr$

$\frac{1}{3r^2} du = dr$

$$\frac{1}{3} \int_{-\pi/2}^{\pi/2} \cos\theta \int_0^8 e^u \, du \, d\theta$$

$$\frac{1}{3} \left(e^u \Big|_0^8 \right) \left(-\sin\theta \Big|_{-\pi/2}^{\pi/2} \right)$$

$$\frac{1}{3} \left(e^8 - e^0 \right) \left(1 - -1 \right) = \boxed{\frac{2}{3}\left(e^8 - 1\right)}$$