Your Name

Your Signature

Student ID #

Your TA's name

Your Quiz Section Label and Time

Problem	Points	Possible
1		10
2		8
3		6
4		10
5		16
Total		50

- No books allowed. You may use a scientific calculator and one $8\frac{1}{2} \times 11$ sheet of notes.
- Do not share notes.
- In order to receive credit, you must show your work and explain your reasoning (except on the "short answer" questions).
- **Place a box around $\boxed{\textbf{YOUR FINAL ANSWER}}$ to each question.**
- If you need more room, use the backs of the pages and indicate to the grader where to find your work.
- Raise your hand if you have a question or need more paper.

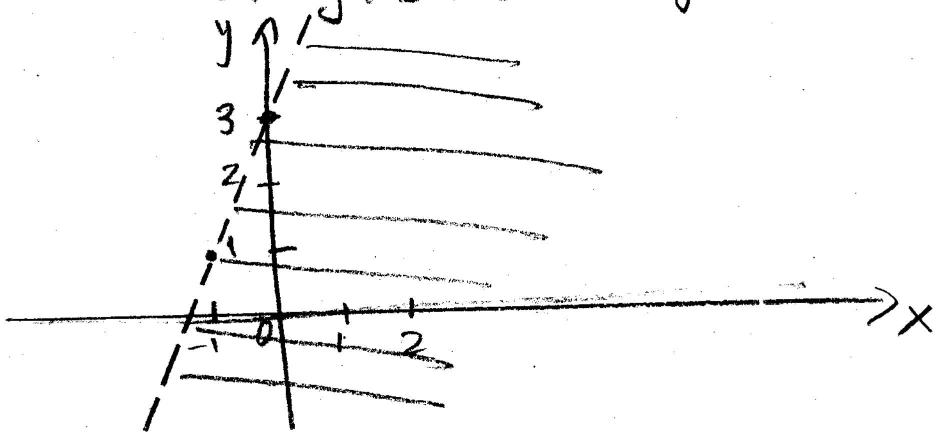
Don't open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!

1. ($10 = 2 + 3 + 5$ **points**) Consider the function $f(x, y) = \ln(2x - y + 3)$.

(a) Find and sketch the domain of this function.

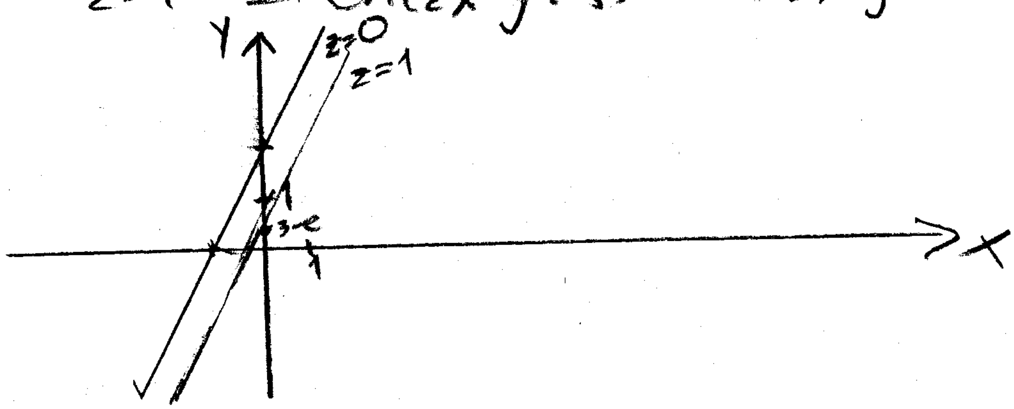
$2x - y + 3 > 0 \Rightarrow y < 2x + 3$



(b) Sketch two level curves of this function: one corresponding to $z = 0$ and another one to $z = 1$.

$z = 0 \Rightarrow \ln(2x - y + 3) = 0 \Rightarrow 2x - y + 3 = 1 \Rightarrow y = 2x + 2$

$z = 1 \Rightarrow \ln(2x - y + 3) = 1 \Rightarrow 2x - y + 3 = e \Rightarrow y = 2x + (3 - e)$



(c) Find the linear approximation for f at $(0, 2)$ and use it to estimate the value $f(0.05, 1.93)$.

$$f_x = \frac{2}{2x - y + 3} \qquad f_y = \frac{-1}{2x - y + 3}$$

$$\Rightarrow f_x(0,2) = \frac{2}{-2+3} = 2 \qquad f_y(0,2) = \frac{-1}{-2+3} = -1$$

$$f(0,2) = \ln(-2+3) = 0$$

$$\Rightarrow \boxed{L(x,y) = 2(x-0) - 1(y-2)} \text{ and}$$

$$f(0.05, 1.93) \approx 2(0.05 - 0) - (1.93 - 2) = 0.1 + 0.07 = \boxed{0.17}$$

2. **(8 points)** Find and classify all critical points of the function $f(x,y) = x^3 + y^2 - 2xy$.

$$\left.\begin{aligned} f_x &= 3x^2 - 2y = 0 \\ f_y &= 2y - 2x = 0 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} 2y &= 3x^2 \\ 2y &= 2x \end{aligned}\right\} \Rightarrow \begin{aligned} 3x^2 - 2x = 0 \\ x(3x-2) = 0 \end{aligned}$$

$$\Rightarrow x = 0 \text{ or } x = \frac{3}{2}$$

$$y = 0 \qquad y = 3/2$$

So there are two cr. pts. $(0,0)$ and $(\frac{3}{2}, \frac{3}{2})$.

$$\begin{aligned} f_{xx} &= 6x & f_{xy} &= -2 \\ f_{yx} &= -2 & f_{yy} &= 2 \end{aligned} \Rightarrow D(x,y) = \begin{vmatrix} 6x & -2 \\ -2 & 2 \end{vmatrix} = 12x - 4$$

At $(0,0)$: $D(0,0) = -4 < 0 \Rightarrow$ $\boxed{(0,0) \text{ is a saddle pt}}$

At $(\frac{3}{2}, \frac{3}{2})$: $D(\frac{3}{2}, \frac{3}{2}) = 18 - 4 > 0$

$f_{xx}(\frac{3}{2}, \frac{3}{2}) = 6 \cdot \frac{3}{2} > 0 \Rightarrow$ $\boxed{(\frac{3}{2}, \frac{3}{2}) \text{ is a pt of local min}}$

3. **(6 points)** For each of the following statements circle the correct answer. *No explanation of answers is needed for this problem. Be sure to explain your answers on other problems!*

   (a) The osculating plane to the curve $\mathbf{r}(t) = \langle t^3 + e^{2t}, 0, \cos(t^4) \rangle$ at the point $(1, 0, 1)$ is **the xy-plane**; **the xz-plane**; **the yz-plane**; **the $x = z$ plane**.

   (b) Let $a_N(t)$ and $a_T(t)$ be the normal and tangential components of acceleration of a certain vector function $\mathbf{r}(t)$.
   Which of the two of them can be negative? **both**; **only a_N**; **only a_T**.

   (c) Let $z = f(x,y)$ be a function of two variables such that $f_x(0,0) = f_y(0,0) = 0$, $f_{xx}(0,0) > 0$ and $f_{yy}(0,0) < 0$. Then $(0,0)$ is
   a saddle point of f; **a local minimum of f**; **a local maximum of f**.

$$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \underbrace{f_{xx}}_{>0} \underbrace{f_{yy}}_{<0} - \underbrace{(f_{xy})^2}_{\geq 0} < 0$$

4. **(10=4+6 points)** All the parts of this problem concern the vector function $\mathbf{r}(t)$ that satisfies the following conditions: the acceleration is $\mathbf{a}(t) = \langle \frac{1}{(t+1)^2}, 0, e^{-t} \rangle$ and the initial position and velocity are given by $\mathbf{r}(0) = \langle 0, 2, 0 \rangle$ and $\mathbf{v}(0) = \langle 0, 1, -1 \rangle$.

(a) Compute the normal component of the acceleration vector at $t = 0$.

$\vec{r}\,'(0) = \vec{v}(0) = \langle 0, 1, -1 \rangle$

$\vec{r}\,''(0) = \vec{a}(0) = \langle \frac{1}{1^2}, 0, e^{-0} \rangle = \langle 1, 0, 1 \rangle$

$\Rightarrow |r'(0)| = \sqrt{2}$ and $\vec{r}\,'(0) \times \vec{r}\,''(0) = \begin{vmatrix} i & j & k \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{vmatrix}$

$= \langle 1, -1, -1 \rangle$

$\Rightarrow |\vec{r}\,'(0) \times \vec{r}\,''(0)| = \sqrt{3}$

Thus $a_N(0) = \boxed{\frac{\sqrt{3}}{\sqrt{2}}}$

(b) Find this vector function $\mathbf{r}(t)$.

$\vec{a}(t) = \langle \frac{1}{(t+1)^2}, 0, e^{-t} \rangle \Longrightarrow$

$$\left.\begin{array}{l} \vec{v}(t) = \langle -\frac{1}{t+1} + C_1, C_2, -e^{-t} + C_3 \rangle \\ \vec{v}(0) = \langle 0, 1, -1 \rangle \end{array}\right\} \Rightarrow$$

$C_1 = 1, \quad C_2 = 1, \quad C_3 = 0$

Thus $\vec{v}(t) = \langle 1 - \frac{1}{t+1}, 1, -e^{-t} \rangle \Longrightarrow$

$$\left.\begin{array}{l} \vec{r}(t) = \langle t - \ln|t+1| + K_1, t + K_2, e^{-t} + K_3 \rangle \\ \vec{r}(0) = \langle 0, 2, 0 \rangle \end{array}\right\} \Rightarrow$$

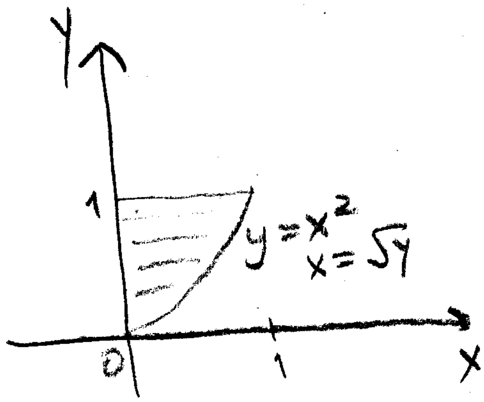
$K_1 = 0, \quad K_2 = 2, \quad K_3 = -1$

$\Rightarrow \boxed{\vec{r}(t) = \langle t - \ln|t+1|, t+2, e^{-t} - 1 \rangle}$

5. **(16=8+8 points)**

(a) Evaluate the integral

$$\int_0^1 \int_{x^2}^1 x \cdot \sin(\pi y^2)\, dy\, dx.$$



$$= \int_0^1 \int_0^{\sqrt{y}} x \cdot \sin(\pi y^2)\, dx\, dy$$

$$= \int_0^1 \left(\frac{x^2}{2} \cdot \sin(\pi y^2) \Big|_{x=0}^{\sqrt{y}} \right) dy$$

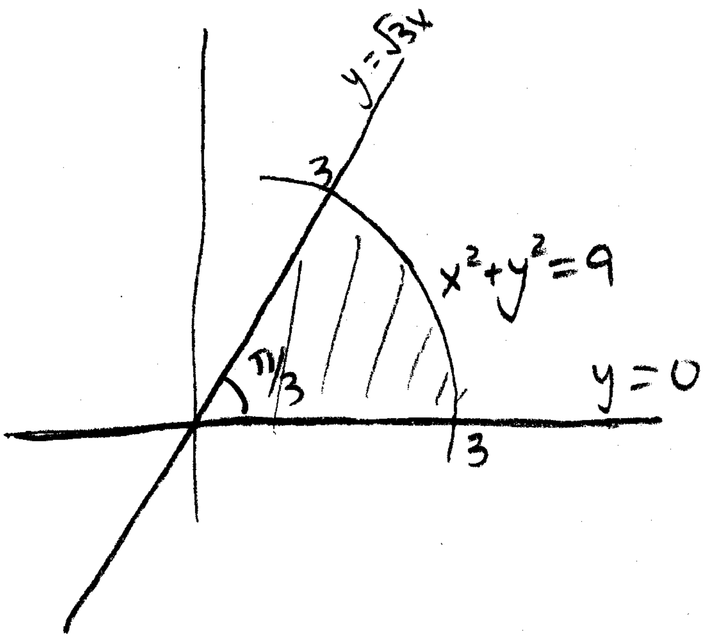
$$= \int_0^1 \frac{y}{2} \cdot \sin(\pi y^2)\, dy$$

$$= \frac{1}{4\pi} \int_0^{\pi} \sin(u)\, du = \frac{1}{4\pi} \left(-\cos u \right)_{u=0}^{\pi}$$

$u = \pi y^2$
$du = 2\pi y\, dy$

$$= \frac{1}{4\pi} \left(1 - (-1) \right) = \boxed{\frac{1}{2\pi}}$$

(b) Calculate the average value of the function $f(x, y) = (x^2 + y^2)^{3/2}$ on the region in the first quadrant bounded by the lines $y = 0$ and $y = \sqrt{3}x$ and the circle $x^2 + y^2 = 9$.



$$\text{Area} = \frac{\pi \cdot 3^2}{6} = \frac{3\pi}{2}$$

$$\iint_D f(x,y)\, dA = \int_0^{\pi/3} \int_0^3 (r^2)^{3/2} \cdot r\, dr\, d\theta$$

$$= \int_0^{\pi/3} \int_0^3 r^4\, dr\, d\theta$$

$$= \left[\frac{r^5}{5} \right]_{r=0}^{3} \cdot \frac{\pi}{3} = \frac{3^5 \pi}{5 \cdot 3} = \frac{81\pi}{5}$$

$$\Rightarrow f_{ave} = \frac{81\pi}{5} \cdot \frac{2}{3\pi} = \boxed{\frac{54}{5}}$$