Your Name

Your Signature

Student ID #

Your TA's name

Your Quiz Section Label and Time

Problem	Points	Possible
1		10
2		8
3		6
4		10
5		16
Total		50

- No books allowed. You may use a scientific calculator and one $8\frac{1}{2} \times 11$ sheet of notes.
- Do not share notes.
- In order to receive credit, you must show your work and explain your reasoning (except on the "short answer" questions).
- **Place a box around $\boxed{\text{YOUR FINAL ANSWER}}$ to each question.**
- If you need more room, use the backs of the pages and indicate to the grader where to find your work.
- Raise your hand if you have a question or need more paper.

Don't open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!

1. **(10=3+3+4 points)** All the parts of this problem concern the vector function $\mathbf{r}(t)$ that satisfies the following conditions: the acceleration is $\mathbf{a}(t) = 6t\mathbf{i} + \mathbf{j} - 12t^2\mathbf{k}$ and the initial position and velocity are given by $\mathbf{r}(0) = 2\mathbf{k}$ and $\mathbf{v}(0) = \mathbf{i} + \mathbf{j}$.

   (a) Write an equation of the normal plane to the curve described by $\mathbf{r}(t)$ at the point where $t = 0$.

   Can take $\vec{r}\,'(0) = \vec{v}(0) = \langle 1, 1, 0 \rangle$ as a normal to this plane, and $\vec{r}(0) = \langle 0, 0, 2 \rangle$ as a pt on this plane.

   Hence, eqn is $1(x-0) + 1(y-0) + 0(z-2) = 0$ or $\boxed{x+y=0}$

   (b) Compute the normal component of acceleration at $t = 0$.

   $$a_N(0) = \frac{|r'(0) \times r''(0)|}{|r'(0)|} = \frac{|\vec{v}(0) \times \vec{a}(0)|}{|\vec{v}(0)|}$$

   $$\left.\begin{array}{l} \vec{v}(0) = \langle 1, 1, 0 \rangle \\ \vec{a}(0) = \langle 0, 1, 0 \rangle \end{array}\right\} \Rightarrow \begin{array}{l} v' \times a' = \langle 0, 0, 1 \rangle \\ |v' \times a'| = 1 \\ |v'| = \sqrt{2} \end{array}$$

   $$\Rightarrow \boxed{a_N = \frac{1}{\sqrt{2}}}$$

   (c) Find this vector function $\mathbf{r}(t)$.

   $$a(t) = \langle 6t, 1, -12t^2 \rangle \Rightarrow$$

   $$\vec{v}(t) = \langle 3t^2 + C_1, t + C_2, -4t^3 + C_3 \rangle$$

   $$\vec{v}(0) = \langle 1, 1, 0 \rangle \Rightarrow C_1 = 1, C_2 = 1, C_3 = 0$$

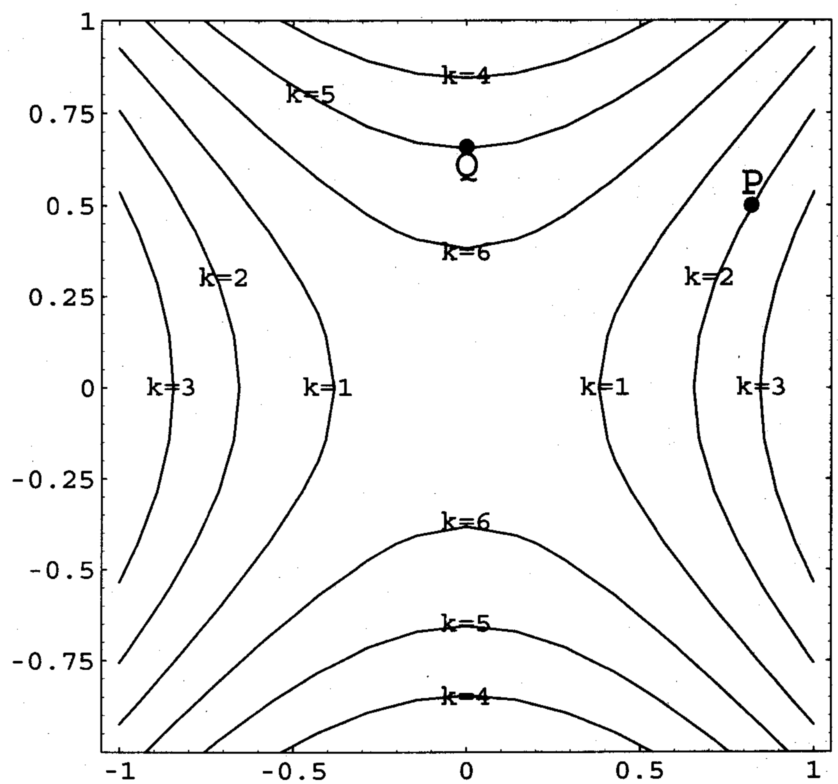
   $$\Rightarrow \vec{v}(t) = \langle 3t^2 + 1, t + 1, -4t^3 \rangle$$

   $$\Rightarrow \vec{r}(t) = \langle t^3 + t + K_1, \frac{t^2}{2} + t + K_2, -t^4 + K_3 \rangle$$

   $$\vec{r}(0) = \langle 0, 0, 2 \rangle \Rightarrow K_1 = K_2 = 0, K_3 = 2$$

   $$\Rightarrow \boxed{\vec{r}(t) = \langle t^3 + t, \frac{t^2}{2} + t, 2 - t^4 \rangle}$$

2. (**8 points**) The level curves of a function $f(x,y)$ are shown below.



Determine whether the following partial derivatives are positive (> 0), negative (< 0) or zero ($= 0$). *No explanation of answers needed for this problem. Be sure to explain your answers on other problems!*

Circle one:

(a) $\frac{\partial f}{\partial x}$ at the point P is $\quad > 0 \quad < 0 \quad = 0$

(b) $\frac{\partial f}{\partial y}$ at the point P is $\quad > 0 \quad < 0 \quad = 0$

(c) $\frac{\partial f}{\partial x}$ at the point Q is $\quad > 0 \quad < 0 \quad = 0$

(d) $\frac{\partial f}{\partial y}$ at the point Q is $\quad > 0 \quad < 0 \quad = 0$

3. (6 = 2 + 2 + 2 **points**) Consider the function $f(x, y) = e^{3x+5y-1}$.

   (a) Calculate the partial derivatives f_x and f_y.

   $f_x = 3e^{3x+5y-1}$
   $f_y = 5e^{5y+5y-1}$

   (b) Write an equation for the tangent plane to the graph of $f(x, y)$ at the point $(2, -1, 1)$.

   $1 \stackrel{?}{=} e^{3\cdot 2+5(-1)+1}$ ✓
   $\Rightarrow$ this pt is on the graph
   $f_x(2,-1) = 3, \; f_y(2,-1) = 5$

   $\boxed{z = 3(x-2) + 5(y+1) + 1}$

   (c) Use the linear approximation for f at $(2, -1)$ to estimate the value $f(1.8, -0.9)$.

   $f(1.8, -0.9) \approx 3(1.8-2) + 5(-0.9+1) + 1$
   $= -0.6 + 0.5 + 1 = \boxed{0.9}$

4. (**10 points**) Find the local maximum and minimum values and saddle points of the function $f(x, y) = (x^2 + y)e^{y/2}$.

Critical pts:

$$\left.\begin{aligned} f_x &= 2x \cdot e^{y/2} = 0 \\ f_y &= e^{y/2} + \frac{1}{2}(x^2+y)\cdot e^{y/2} = \frac{1}{2}e^{y/2}(2+x^2+y) = 0 \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} x&=0 \\ 2+x^2+y&=0 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} x&=0 \\ y&=-2 \end{aligned}\right\}$$

$\Rightarrow (0,-2)$ is the only critical pt

$$f_{xx} = 2e^{y/2} \qquad f_{xy} = 2xe^{y/2}\cdot\frac{1}{2} = xe^{y/2}$$

$$f_{yx} = xe^{y/2} \qquad f_{yy} = \frac{1}{4}e^{y/2}(2+x^2+y) + \frac{1}{2}\cdot e^{y/2} = \frac{1}{4}e^{y/2}(4+x^2+y)$$

$$\Rightarrow D(0,-2) = \begin{vmatrix} 2e^{-1} & 0 \\ 0 & \frac{1}{4}e^{-1}\cdot 2 \end{vmatrix} = e^{-2} > 0$$

and $f_{xx}(0,-2) = 2e^{-1} > 0$

$\Rightarrow$ have a local min.

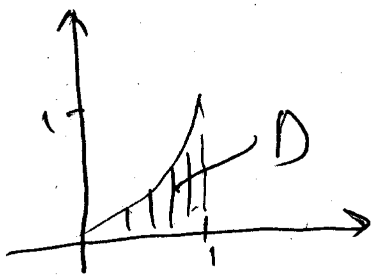
The value of this local min is

$$f(0,-2) = -2\cdot e^{-1} = \boxed{-\frac{2}{e}}$$

5. **(8 points)**

(a) Evaluate the integral

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\, dx\, dy.$$



$x = \sqrt{y}$
$y = x^2$

$$= \int_0^1 \int_0^{x^2} \sqrt{x^3+1}\, dy\, dx$$

$$= \int_0^1 \left(\sqrt{x^3+1} \cdot y \Big|_{y=0}^{x^2} \right) dx$$

$$= \int_0^1 x^2 \cdot \sqrt{x^3+1}\ dx$$

$$= \frac{1}{3}\int_1^2 u^{1/2}\, du = \frac{1}{3} \cdot \frac{2}{3}\ u^{3/2} \Big|_1^2$$

$u = x^3+1$
$du = 3x^2\, dx$

$$= \frac{2}{9}\left(2^{3/2}-1\right) = \boxed{\frac{2}{9}\left(2\sqrt{2}-1\right)}$$

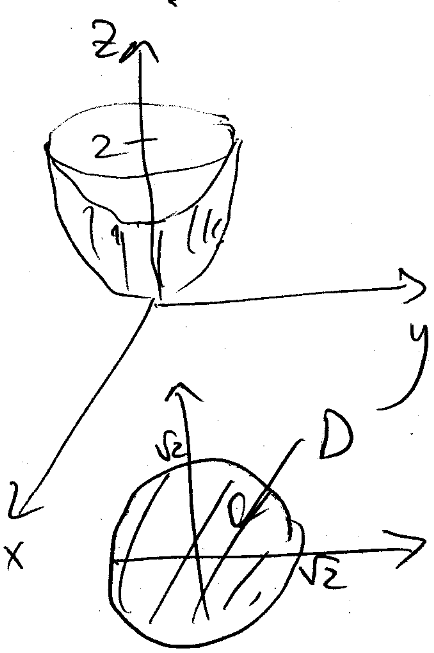
(8 pts) $x=0 \rightsquigarrow u=1$, $x=1 \rightsquigarrow u=2$

(b) Compute the volume of the solid bounded by the paraboloids $z = x^2+y^2$ from below and $z = \frac{x^2}{2}+\frac{y^2}{2}+1$ from above. (**Hint:** draw a picture.)

Intersection:

$$\frac{x^2}{2}+\frac{y^2}{2}+1 = x^2+y^2$$

$$\Rightarrow \frac{x^2+y^2}{2} = 1 \Rightarrow \underbrace{x^2+y^2}_{z} = 2$$



$$V = \iint_D \left(\frac{x^2}{2}+\frac{y^2}{2}+1\right) - (x^2+y^2)\, dA$$

$$= \int_0^{2\pi}\int_0^{\sqrt{2}} \left(1-\frac{r^2}{2}\right) \cdot r\, dr\, d\theta$$

$$= 2\pi \int_0^{\sqrt{2}} \left(r - \frac{r^3}{2}\right) dr = 2\pi \left[\frac{r^2}{2} - \frac{r^4}{8}\right]_0^{\sqrt{2}}$$

$$= 2\pi \left[\frac{2}{2} - \frac{4}{8}\right] = 2\pi \cdot \frac{1}{2} = \boxed{\pi}$$