Math 126 C - Spring 2010
Mid-Term Exam Number Two
May 13, 2010

Name: ___________________________  Student ID no.: ________________

Signature: ___________________________  Section: ____________

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• Complete all questions.

• You may use a scientific calculator during this examination; graphing calculators and other electronic devices are not allowed and should be turned off for the duration of the exam.

• If you use trial-and-error, a guess-and-check method, or numerical approximation when an exact method is available, you will not receive full credit.

• You may use one double-sided, hand-written, 8.5 by 11 inch page of notes.

• Show all work for full credit.

• You have 50 minutes to complete the exam.
1. A particle moves along a curve in the \( xy \)-plane so that its position vector is
\[
\vec{r}(t) = (t + \cos t, t - \sin t)
\]
for \( t \geq 0 \). Assume \( t \) is in seconds, and coordinates are in centimeters.

(a) Find the speed of the particle at time \( t = \pi \).

(b) There are infinitely many times \( t \) when the velocity vector and the acceleration vector for this particle are orthogonal. Give one of these times.
2. Find the curvature of the curve

\[ x = t^2, \quad y = 1 - t, \quad z = 1 - t^2 \]

at the point \( t = 3 \).
3. Let

\[ f(x, y) = \frac{1}{x} + \frac{1}{y} + x + y. \]

(a) Find a point on the surface \( z = f(x, y) \) where the tangent plane is parallel to the plane \( 48x + 6y + 2z = 7 \).

(b) Find and classify all critical points of the surface \( z = f(x, y) \).
4. Let $R$ be the region in the first quadrant of the $xy$-plane bounded by $y = 6 - x$, $y = 6 - 2x$, and the $x$-axis.

Express the volume of three-dimensional space lying above $R$ and below the surface

$$z = xy$$

as one iterated double integral.