Print Your Full Name: Solutions

Signature:

Student ID Number:

Quiz Section:

Instructor's Name:

TA's Name:

Please read these instructions!

1. Your exam contains 8 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. You are allowed a single, double-sided 8.5"x11" handwritten notesheet; the TI-30XIIS calculator; a writing utensil; and an eraser to use on the exam.
3. The exam is worth 50 points. Point values for problems vary and these are clearly indicated. You have 50 minutes for this exam.
4. Make sure to ALWAYS SHOW YOUR WORK CLEARLY. Credit is awarded to work which is clearly shown and legible. Full credit may not be awarded if work is unclear or illegible.
5. For problems that aren't sketches, place a $\boxed{\text{box around your final answer}}$ to each question.
6. If you need extra space, use the last two pages of the exam. Clearly indicate on that there is more work located on the last pages, and indicate on those pages the related problem number.
7. Unless otherwise instructed, always give your answers in exact form. For example, 3π, $\sqrt{2}$, and $\ln(2)$ are in exact form; the corresponding approximations 9.424778, 1.4142, and 0.693147 are NOT in exact form.
8. Credit is awarded for correct use of techniques or methods discussed in class thus far. Partial credit may be awarded as earned. No credit is awarded for use of methods that are learned later in the course.

Problem	Total Points
1	10
2	10
3	10
4	10
5	10
Total	50

1. (10 points) Find all critical points of the function $f(x,y) = 2xye^{-x^2}$. Classify each point as corresponding to a local maximum value, local minimum value, or a saddle point. You do not need to find the z-coordinate of each point.

$f_x = 2xy \cdot e^{-x^2} \cdot -2x + e^{-x^2} \cdot 2y$ (+2) $\rightarrow$ $x=0$, $2y=0 \longrightarrow y=0$

$= 2e^{-x^2}(-2x^2y + y) = 0$

$(0,0)$ (+2)

$f_y = 2xe^{-x^2}$ (+2) $= 0 \longrightarrow x=0$

$f_{yx} = 2x \cdot e^{-x^2} \cdot -2x + e^{-x^2} \cdot 2 = 2e^{-x^2}(-2x^2+1)$

$f_{yy} = 0$

$\Rightarrow D = f_{xx}f_{yy} - f_{xy}^2 = -\left(2e^{-x^2}(-2x^2+1)\right)^2$ (+2)

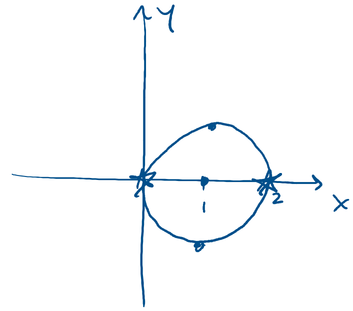
$D(0,0) < 0 \Rightarrow \boxed{\text{saddle point at } (0,0)}$

(+1) (+1)

2. (10 points) Find the absolute maximum and minimum values of $f(x, y) = 2x^2 - 4x + 3y^2 + 2$ on the set $D = \{(x, y) \mid (x-1)^2 + y^2 \leq 1\}$. You must use methods discussed in class to justify your answer.

$f_x = 4x - 4 = 0 \rightarrow x = 1$ crit. pt. $(1,0)$

$f_y = 6y = 0 \rightarrow y = 0$ +2



Boundary: $(x-1)^2 + y^2 = 1$

$y^2 = 1 - (x-1)^2$

$\Rightarrow \quad 2x^2 - 4x + 3(1 - (x-1)^2) + 2 = g(x) \qquad 0 \leq x \leq 2$

$2x^2 - 4x + 3 - 3(x-1)^2 + 2 = g(x)$

$0 = 4x - 4 - 6(x-1) = g'(x)$

+5

$-2x + 2 = 0 \rightarrow x = 1$ Points: $(1, 1)$

$(1, -1)$

End points: $(0,0)$ $(2,0)$

+1

Check:

$f(0,0) = 2$

$f(2,0) = 2$

$\boxed{\begin{array}{l} f(1,0) = 0 \leftarrow \text{abs min value} \\ f(1,1) = f(1,-1) = 3 \leftarrow \text{abs max value} \end{array}}$ +1 +1

3. Let $f(x, y) = \sqrt{38 - x^2 - 4y^2}$.

   (a) (8 points) Find the linear approximation of $f(x, y)$ at the point $(5, 1)$.

$$f_x = \frac{1}{2\sqrt{38-x^2-4y^2}} \cdot -2x \quad (+2) \qquad f_x(5,1) = \frac{-5}{3} \quad (+1)$$

$$f_y = \frac{1}{2\sqrt{38-x^2-4y^2}} \cdot -8y \quad (+2) \qquad f_y(5,1) = \frac{-4}{3} \quad (+1)$$

$$f(5,1) = \sqrt{38-25-4} = \sqrt{9} = 3 \quad (+1)$$

$$\boxed{L(x,y) = \frac{-5}{3}(x-5) - \frac{4}{3}(y-1) + 3}$$

(+1)

   (b) (2 points) Use part (a) to find an approximate value of $f(5.06, 1.08)$. Write your answer as a single number rounded to two decimal places.

$$f(5.06, 1.08) \approx L(5.06, 1.08)$$

$$= \frac{-5}{3}(0.06) - \frac{4}{3}(0.08) + 3 = \boxed{2.79}$$

(+2)

4. Parts (a) and (b) below are unrelated.

   (a) (5 points) Compute the integral $\iint_R \frac{3xy^2}{x^2+1}dA$, where $R = \{(x,y) \mid 0 \le x \le 1, -2 \le y \le 2\}$.

$$\int_0^1\int_{-2}^{2} \frac{3xy^2}{x^2+1}\,dy\,dx \quad +2$$

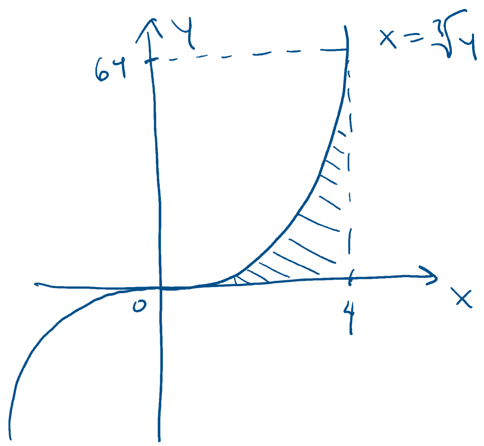
$$= \int_0^1 \left[\frac{x\,y^3}{x^2+1}\right]_{-2}^{2} dx$$

$$= \int_0^1 \frac{16x}{x^2+1}\,dx \quad +2 \qquad u = x^2+1, \quad du = 2x\,dx, \quad \tfrac{1}{2}du = x\,dx$$

$$= 8\int_1^2 \frac{1}{u}\,du$$

$$= 8\left[\ln(u)\right]_1^2 = \boxed{8\ln 2} \quad +1$$

   (b) (5 points) Reverse the order of integration, but **do not evaluate the integral**: $\int_0^{64}\int_{\sqrt[3]{y}}^{4} 6e^{x^4}\,dx\,dy$.



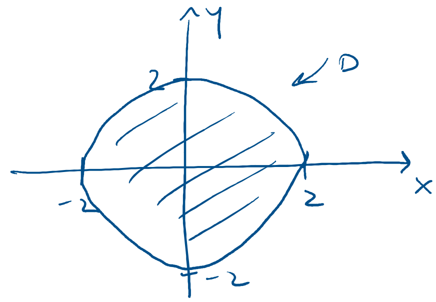
$x = \sqrt[3]{y} \rightarrow x^3 = y$

$$\boxed{\int_0^4\int_0^{x^3} 6e^{x^4}\,dy\,dx}$$

+1 +1 +1 +1 +1

5. (10 points) Use polar coordinates to find the volume of the solid bounded by the paraboloid $z = 8 - x^2 - 3y^2$ and the hyperbolic paraboloid $z = x^2 - y^2$.

$$8 - x^2 - 3y^2 = x^2 - y^2$$
$$8 = 2x^2 + 2y^2$$
$$4 = x^2 + y^2$$



$$V = \iint_D \left(8 - x^2 - 3y^2 - (x^2 - y^2)\right) dA$$

+2

$$= \iint_D \left(8 - 2x^2 - 2y^2\right) dA$$

$$= \boxed{\int_0^{2\pi}\int_0^{2}} \overbrace{\underline{(8 - 2r^2)}\,\underline{r\,dr}}^{8r - 2r^3}\,\underline{d\theta}$$

+2 +2 +2

$$= \int_0^{2\pi} \left[4r^2 - \frac{1}{2}r^4\right]_0^2 d\theta$$

$$= \int_0^{2\pi} (16 - 8)\, d\theta = \int_0^{2\pi} 8\, d\theta = 8 \cdot 2\pi = \boxed{16\pi}$$

+2

Extra scratch paper.

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