1. **[7 points]** A particle moves with position vector $\mathbf{r}(t) = \langle 8t+1, t^2-4t, \frac{1}{2}t^3 \rangle$.

   Find the tangential and normal components of acceleration of the particle at time $t = 2$.

$\vec{r}\,'(t) = \langle 8, 2t-4, \frac{3}{2}t^2 \rangle$ $\quad$ $\vec{r}\,'(2) = \langle 8, 0, 6 \rangle$

$\vec{r}\,''(t) = \langle 0, 2, 3t \rangle$ $\quad$ $\vec{r}\,''(2) = \langle 0, 2, 6 \rangle$

$\vec{r}\,'(2) \cdot \vec{r}\,''(2) = 36$

$\vec{r}\,'(2) \times \vec{r}\,''(2) = \langle -12, -48, 16 \rangle$

$|\vec{r}\,'(2)| = 10$

$|\vec{r}\,'(2) \times \vec{r}\,''(2)| = 52$

$$\boxed{\begin{array}{l} a_T = \frac{36}{10} = 3.6 \\ a_N = \frac{52}{10} = 5.2 \end{array}}$$

2. **[8 points]** Let $\mathcal{S}$ be the surface

$$xz + \sqrt{z} - 2xy = 6.$$

   Find the equation of the plane tangent to $\mathcal{S}$ at the point $(2, 1, 4)$.

$\frac{\partial}{\partial x}$ $\qquad$ $\frac{\partial}{\partial y}$

$z + x\frac{\partial z}{\partial x} + \frac{1}{2\sqrt{z}}\frac{\partial z}{\partial x} - 2y = 0$ $\qquad$ $x\frac{\partial z}{\partial y} + \frac{1}{2\sqrt{z}}\frac{\partial z}{\partial y} - 2x = 0$

$\downarrow (2,1,4)$ $\qquad$ $\downarrow (2,1,4)$

$4 + 2\frac{\partial z}{\partial x} + \frac{1}{4}\frac{\partial z}{\partial x} - 2 = 0$ $\qquad$ $2\frac{\partial z}{\partial y} + \frac{1}{4}\frac{\partial z}{\partial y} - 4 = 0$

$\frac{9}{4}\frac{\partial z}{\partial x} = -2$ $\qquad$ $\frac{9}{4}\frac{\partial z}{\partial y} = 4$

$\frac{\partial z}{\partial x} = \frac{-8}{9}$ $\qquad$ $\frac{\partial z}{\partial y} = \frac{16}{9}$

$$\boxed{z = \frac{-8}{9}(x-2) + \frac{16}{9}(y-1) + 4}$$

3. **[15 points]** Consider the function $f(x, y) = xy^2 - 2xy + x^2$.

Find all critical points of f, **and classify them** as local minima, local maxima, or saddle points.

$f_x(x,y) = y^2 - 2y + 2x = 0$

$f_y(x,y) = 2xy - 2x = 0 \rightarrow 2x(y-1) = 0$

$x = 0$ or $y = 1$

$y^2 - 2y = 0$

$y(y-2) = 0$

$y = 0$ or $y = 2$

$1 - 2 + 2x = 0$

$x = \frac{1}{2}$

Three critical points: $(0,0)$, $(0,2)$, and $(\frac{1}{2}, 1)$

$f_{xx}(x,y) = 2$

$f_{yy}(x,y) = 2x$

$f_{xy}(x,y) = 2y - 2$

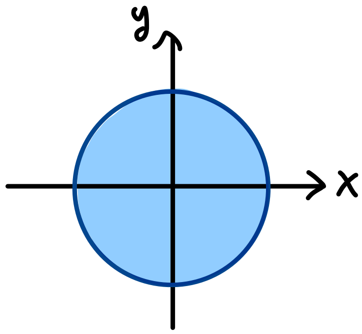
$D(0,0) = 0 - (-2)^2 = -4 < 0$. saddle point @ $(0,0)$

$D(0,2) = 0 - (2)^2 = -4 < 0$ saddle point @ $(0,2)$

$D(\frac{1}{2}, 1) = 2 \cdot 1 - 0^2 = 2 > 0$ local min @ $(\frac{1}{2}, 1)$

4. **[15 points]** Let f be the function $f(x, y) = xy^2 + x^2$, and let $\mathcal{D}$ be closed disc of radius 1 centered at the origin.

Find the absolute minimum and maximum values of f on $\mathcal{D}$.



Critical points

$f_x(x,y) = y^2 + 2x = 0$

$f_y(x,y) = 2xy = 0 \rightarrow x = 0 \rightarrow y = 0$ or $y = 0 \rightarrow x = 0$

So, just $(0,0)$

Boundary: $x^2 + y^2 = 1 \rightarrow y^2 = 1 - x^2$

$f(x,y) = x(1 - x^2) + x^2 = x - x^3 + x^2$, domain $-1 \leq x \leq 1 \rightarrow (-1,0)$ & $(1,0)$

$f'(x) = 1 - 3x^2 + 2x = 0$

$(3x+1)(-x+1) = 0$

$x = \frac{-1}{3}$ $\qquad$ $x = 1$

$y = \pm\sqrt{\frac{8}{9}}$ $\qquad$ $y = 0$

Points to check

$f(0,0) = 0$

$f(-1,0) = 1$

$f(1,0) = 1$

$f\left(\frac{-1}{3}, \frac{2\sqrt{2}}{3}\right) = \frac{-5}{27}$

$f\left(\frac{-1}{3}, \frac{-2\sqrt{2}}{3}\right) = \frac{-5}{27}$

Max 1

Min $\frac{-5}{27}$

5. **[8 points]** Evaluate $\int_0^5 \int_3^4 (2xy^2 + e^y)\,dy\,dx$.

$$= \int_3^4 \int_0^5 (2xy^2 + e^y)\,dx\,dy$$

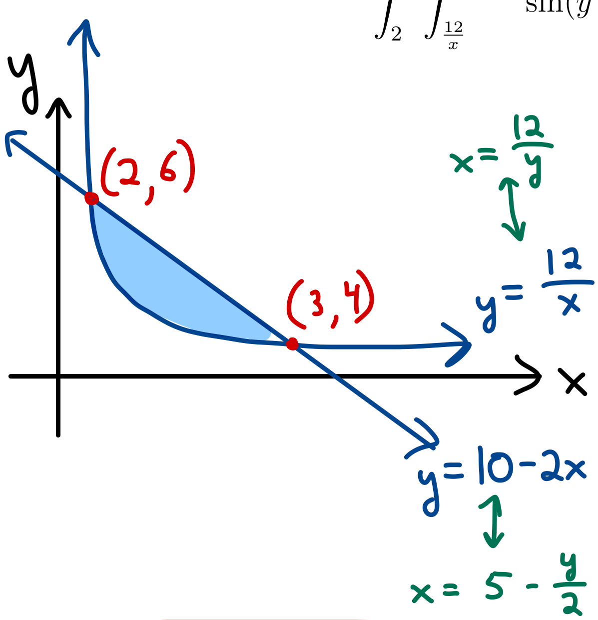
$$= \int_3^4 \left(x^2y^2 + xe^y\right)\Big]_{x=0}^{x=5} dy = \int_3^4 \left(25y^2 + 5e^y\right)dy$$

$$= \left(\frac{25}{3}y^3 + 5e^y\right)\Big]_3^4 = \left(\frac{25}{3}64 + 5e^4\right) - \left(\frac{25}{3}27 - 5e^3\right)$$

$$= \boxed{\frac{925}{3} + 5e^4 - 5e^3}$$

6. **[7 points]** Rewrite the following integral after reversing the order of integration.
Do not try to evaluate the integral! Just reverse the order of integration.

$$\int_2^3 \int_{\frac{12}{x}}^{10-2x} \sin(y^2)\,dy\,dx$$



Intersection of $y = \frac{12}{x}$, $y = 10-2x$.

$$\frac{12}{x} = 10 - 2x$$
$$12 = 10x - 2x^2$$
$$x^2 - 5x + 6 = 0$$
$$(x-3)(x-2) = 0$$
$$x = 2 \text{ or } 3$$

$$\boxed{\int_4^6 \int_{\frac{12}{y}}^{5-\frac{y}{2}} \sin(y^2)\,dx\,dy}$$