1. **[5 points per part]** For this problem, consider the points

$$A = (2,3,3) \qquad B = (1,1,1) \qquad C = (1,4,-3).$$

(a) Compute the angle $\angle ABC$.

$\overrightarrow{BA} = \langle 1,2,2 \rangle$

$\overrightarrow{BC} = \langle 0,3,-4 \rangle$

$|\overrightarrow{BA}| = \sqrt{1+4+4} = 3$

$|\overrightarrow{BC}| = \sqrt{9+16} = 5$

$\overrightarrow{BA} \cdot \overrightarrow{BC} = 0+6-8 = -2$

$\overrightarrow{BA} \cdot \overrightarrow{BC} = |\overrightarrow{BA}||\overrightarrow{BC}| \cos\theta$

$-2 = 3\,5 \cos\theta$

$\theta = \boxed{\cos^{-1}\left(\frac{-2}{15}\right)} \approx 1705 \text{ rad} \approx 977^\circ$

(b) Find the equation of the plane containing A, B, and C.

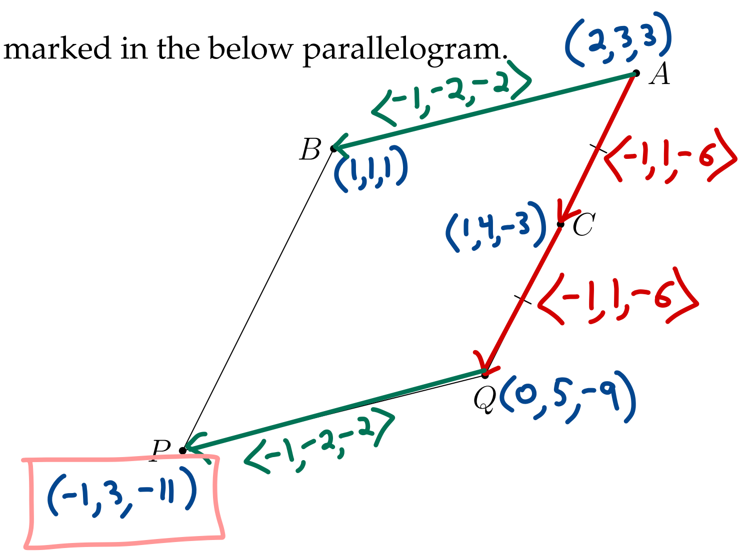
$\overrightarrow{BA} \times \overrightarrow{BC} = \underbrace{\langle -14, 4, 3 \rangle}_{\text{normal vector}}$

$-14(x-2) + 4(y-3) + 3(z-3) = 0$

or $\boxed{-14x + 4y + 3z = -7}$

(c) Find the coordinates of the point P marked in the below parallelogram.

(C is the midpoint of AQ.)



2. **[2 points per part]** For each of the following objects, figure out how they intersect.

Circle one option. You do not need to show work on this problem.

(a) The line $x = t, y = 3t, z = 1 - 4t$ and the plane $x + 3y - 4z = 7$.

a point a line a plane no intersection

Line is normal to plane, so they intersect at a point

(b) The line $x = t, y = 2t, z = 3t$ and the plane $x + y - z = 1$.

a point a line a plane no intersection

$t + 2t - 3t = 1 \rightarrow 0 = 1$, no solutions

(c) The planes $2x + 4y + 6z = 2$ and $3x + 6y + 9z = 3$.

a point a line a plane no intersection

These are the same plane!

(d) The planes $x + 4y - 2z = 1$ and $x + 4y + 2z = 7$.

a point a line a plane no intersection

These planes are not parallel, so they intersect in a line

3. *You do not need to show work on parts (a) and (b). Please show work on part (c).*

(a) **[3 points]** Give an example of a vector **a** such that $\mathbf{a} \cdot \mathbf{a} = 7$.

Any vector where $|\vec{a}| = \sqrt{7}$, eg $\boxed{\langle \sqrt{7}, 0, 0 \rangle}$

(b) **[3 points]** Give an example of a vector **a** such that $\text{comp}_{\mathbf{a}} \langle 1, 2, 3 \rangle = -2$.

One way any vector that points along the negative y-axis, like $\boxed{a = \langle 0, -1, 0 \rangle}$

(c) **[5 points]** Give an example of two nonzero vectors **a** and **b** such that $|\mathbf{a} \times \mathbf{b}| = -\mathbf{a} \cdot \mathbf{b}$.

$|\vec{a}||\vec{b}| \sin\theta = -|\vec{a}||\vec{b}| \cos\theta$

$\tan\theta = -1 \rightarrow \theta = \frac{3\pi}{4}$

Any vectors that make an angle of $\frac{3\pi}{4}$,

eg $\boxed{\langle 1, 0, 0 \rangle \text{ and } \langle -1, 1, 0 \rangle}$

positive

$\vec{a} \cdot \vec{b}$ is negative

So θ is obtuse

4. **[8 points]** Find the equation of the ellipsoid which is centered at the origin and contains the points $(3,8,0)$, $(5,0,0)$, and $(2,1,1)$.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$$\frac{5^2}{a^2}=1 \rightarrow a^2=5^2$$

$$\frac{3^2}{5^2}+\frac{8^2}{b^2}=1 \rightarrow \frac{64}{b^2}=\frac{16}{25} \rightarrow b^2=10^2$$

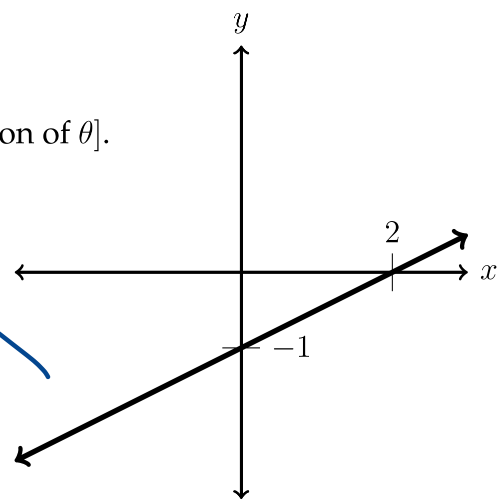
$$\frac{4}{5^2}+\frac{1}{10^2}+\frac{1}{c^2}=1 \rightarrow \frac{1}{c^2}=\frac{83}{100} \rightarrow c^2=\frac{100}{83}$$

$$\boxed{\frac{x^2}{5^2}+\frac{y^2}{10^2}+\frac{83z^2}{100}=1}$$

5. **[6 points]** Consider the line in the following graph.

Convert the equation for this line to polar form.

Write your final answer in the form $r=$ [some function of θ].



$$y=\frac{1}{2}x-1$$

$$r\sin\theta=\frac{1}{2}r\cos\theta-1$$

$$r\sin\theta-\frac{1}{2}r\cos\theta=-1$$

$$\boxed{r=\frac{-1}{\sin\theta-\frac{1}{2}\cos\theta}}$$

6. **[6 points per part]** The force exerted on an 8-kg cat at time t is given by the vector function

$$\vec{F}(t) = \langle 16t, 24, 8\sqrt{t+7}\rangle \quad \text{Newtons.}$$

At time $t = 0$, the cat is at the point $(1, 2, 3)$. At time $t = 2$, the cat is at rest.

(a) Find parametric equations for the line tangent to the cat's path at time $t = 0$.

$$\text{Acceleration} = \vec{a}(t) = \frac{\vec{F}}{m} = \left\langle 2t, 3, \sqrt{t+7} \right\rangle$$

$$\vec{v}(t) = \left\langle t^2 + C_1, 3t + C_2, \tfrac{2}{3}(t+7)^{3/2} + C_3 \right\rangle$$

$$\vec{v}(2) = \left\langle 4 + C_1, 6 + C_2, \tfrac{2}{3}(27) + C_3 \right\rangle = \langle 0, 0, 0 \rangle \to C_1 = -4,\ C_2 = -6,\ C_3 = -18$$

$$\vec{v}(0) = \left\langle C_1, C_2, \tfrac{2}{3}(7)^{3/2} + C_3 \right\rangle = \underbrace{\left\langle -4, -6, \tfrac{2}{3}(7)^{3/2} - 18 \right\rangle}_{\text{direction}}$$

$$\boxed{\begin{aligned} x &= 1 - 4t \\ y &= 2 - 6t \\ z &= 3 + \left(\tfrac{2}{3}(7)^{3/2} - 18\right)t \end{aligned}}$$

(b) Find the tangential component of acceleration of the cat at time $t = 0$.

$$\vec{v}(0) = \left\langle -4, -6, \tfrac{2}{3}(7)^{3/2} - 18 \right\rangle, \quad \vec{a}(0) = \left\langle 0, 3, \sqrt{7} \right\rangle$$

$$a_T = \frac{\vec{v}(0) \cdot \vec{a}(0)}{|\vec{v}(0)|} = \frac{-18 + \tfrac{2}{3}(49) - 18\sqrt{7})}{\left|\left\langle -4, -6, \tfrac{2}{3}(7)^{3/2} - 18 \right\rangle\right|}$$

$$= \boxed{\frac{-18 + \tfrac{2}{3}(49) - 18\sqrt{7}}{\sqrt{16 + 36 + \left(\tfrac{2}{3}(7)^{3/2} - 18\right)^2}}}$$