MATH 126 C
Exam I
Winter 2019

Name ______________________________

Student ID #______________ Section ______________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:______________________________

- Your exam should consist of this cover sheet, followed by 6 problems on 5 pages. Check that you have a complete exam.
- Pace yourself. You have 50 minutes to complete the exam and there are 5 pages. Try not to spend more than 10 minutes on each page.
- Unless otherwise indicated, **show all your work and justify your answers**.
- Unless otherwise indicated, your answers should be exact values rather than decimal approximations. (For example, $\frac{\pi}{4}$ is an exact answer and is preferable to its decimal approximation 0.7854.)
- You may use a **TI 30XII S** calculator and one 8.5×11-inch sheet of handwritten notes. **All other calculators, electronic devices, and sources are forbidden**.
- **Do not write within one centimeter of the edge of the page.** Your exam will be scanned for grading.
- If you need more room, ask your TA for extra paper, put your name on it, and **tell the grader where to look for your solution**.
- The use of headphones or earbuds during the exam is not permitted.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.
- You are not allowed to use your phone for any reason during this exam. **Turn your phone off and put it away for the duration of the exam**.

GOOD LUCK!

1. (7 points) Consider the surface given by the equation

$$4x^2 - 100y^2 - 25z^2 + 100 = 0.$$

(a) Give the intercepts of the surface with each of the coordinate axes. If no such intercepts exist, write NONE.

i. x-intercept(s): ____________________

ii. y-intercept(s): ____________________

iii. z-intercept(s): ____________________

(b) Identify the trace of the surface in the given plane.

i. $x = k$

ANSWER: (circle one) circle ellipse hyperbola parabola none of these

ii. $y = k$

ANSWER: (circle one) circle ellipse hyperbola parabola none of these

iii. $z = k$

ANSWER: (circle one) circle ellipse hyperbola parabola none of these

(c) Identify the surface. (You do not need to show any work.)
Choose your answer from the following list:

elliptic cylinder	parabolic cylinder	hyperbolic cylinder
paraboloid	ellipsoid	hyperbolic paraboloid
cone	hyperboloid of one sheet	hyperboloid of two sheets

2. (10 points)

   (a) For some real numbers a and b, the vector $\mathbf{n} = a\mathbf{i} + b\mathbf{j} - 4\mathbf{k}$ is orthogonal to the plane $z = 3x$.

      i. What are a and b?

      ii. What is $\mathbf{proj_k n}$?

   (b) What is the angle between the plane $z = 3x$ and the xy-plane?

   (c) Give parametric equations for the curve that is the intersection of the plane $z = 3x$ with the cylinder $x^2 + y^2 = 1$.

3. (10 points) Indicate whether each of the following is true (T) or false (F). Circle your answer. No justification for your answer is needed.

   (a) **T** **F** Two planes in $\mathbb{R}^3$ that are not parallel must intersect.

   (b) **T** **F** Given a plane $\mathscr{P}$, there is exactly one plane perpendicular to $\mathscr{P}$.

   (c) **T** **F** The plane that contains the point $(1, 1, 1)$ and is orthogonal to the vector $\mathbf{n}_1 = \langle 1, 2, -3 \rangle$ is the same as the plane that contains the point $(3, 0, 1)$ and is orthogonal to the vector $\mathbf{n}_2 = \langle -2, -4, 6 \rangle$.

   (d) **T** **F** The triangle with vertices $P(4, 2, -1)$, $Q(6, 3, -3)$, and $R(8, 3, -1)$ is a right triangle.

   (e) **T** **F** The plane $6x - 4y + 2z = 4$ contains the line defined by the vector function $\mathbf{r}(t) = \langle -3t, 2t + 7, 6 - t \rangle$.

4. (7 points) The position function of a particle is given by $\mathbf{r}(t) = \left\langle 3\cos t, t^2 - t, 3\sin t \right\rangle$. (Here t is in seconds and x, y and z are measured in feet.) Compute the minimum **speed** of the particle.

5. (8 points) A line, ℓ, passes thru $(1, 2, 2)$ and the *center* of the sphere, S, given by

$$x^2 + y^2 + z^2 - 6z = 27.$$

Find all points of intersection (x, y, z) of the line, ℓ, and the sphere, S.

6. (8 points) Find the **radius of curvature** of the curve

$$x = t\cos t, y = t\sin t$$

at the point $(-\pi, 0)$.