1. **[6 points per part]** Here are some short, unrelated plane problems.

   (a) Find the plane through the points $\underbrace{(1,2,3)}_{A}$, $\underbrace{(2,3,4)}_{B}$, and $\underbrace{(4,6,8)}_{C}$.

$\vec{AB} = \langle 1,1,1 \rangle$

$\vec{AC} = \langle 3,4,5 \rangle$

$\vec{AB} \times \vec{AC} = \langle 1,-2,1 \rangle$ → normal vector of plane

So: $(x-1) - 2(y-2) + (z-3) = 0$

or: $\boxed{x - 2y + z = 0}$

   (b) Find the line of intersection of the planes $y = 2x + 3$ and $4x - 5y + 2z = 7$.

   Write your answer in parametric form.

Let's arbitrarily set $x = t$.

So $y = 2t + 3$,

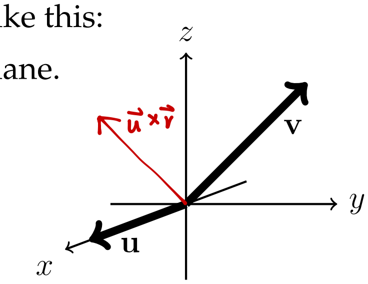
and $2z = 7 - 4x + 5y$

$z = \frac{7}{2} - 2t + \frac{5}{2}(2t+3)$

$z = 11 + 3t$ →

$$\boxed{\begin{aligned} x &= t \\ y &= 3 + 2t \\ z &= 11 + 3t \end{aligned}}$$

2. **[2 points per part]** Suppose **u** and **v** are 3D vectors that look like this: That is, **u** points along the positive x-axis, and **v** is in the yz-plane. (You don't have to show work on this problem.)



Identify the following values as positive, negative, or zero. (Circle your answer.)

(a) $\mathbf{u} \cdot \mathbf{v}$: positive negative $\boxed{\text{zero}}$

(b) The x-component of $\mathbf{u} \times \mathbf{v}$: positive negative $\boxed{\text{zero}}$

(c) The y-component of $\mathbf{u} \times \mathbf{v}$: positive $\boxed{\text{negative}}$ zero

(d) The z-component of $\mathbf{u} \times \mathbf{v}$: $\boxed{\text{positive}}$ negative zero

3. **[7 points]** Graph the polar curve $r = \dfrac{1}{\sin(\theta) - \cos(\theta)}$. Please label the scales on your axes.

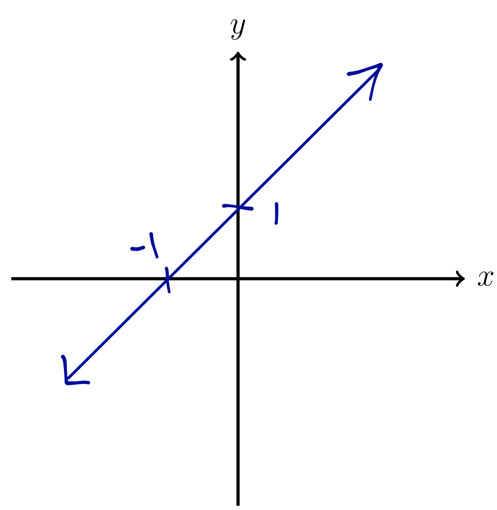
(Hint: This is a graph you've seen before. Don't *just* draw a picture; do some algebra first.)

$r(\sin\theta - \cos\theta) = 1$

$r\sin\theta - r\cos\theta = 1$

$y - x = 1$

$y = x + 1 \rightarrow$ hey, that's a line!



4. **[8 points]** You just got a job at Quadricorp, the company that builds quadric surfaces.

   Your first assignment: make a hyperboloid of **two sheets** following these specifications:

   - The hyperboloid "points" along the z-axis.
   - The hyperboloid goes through the points $(0, 0, 2)$, $(3, 0, 4)$, and $(6, 4, 8)$.

   Give the equation of this hyperboloid.

$$\frac{-x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$-0 - 0 + \frac{4}{c^2} = 1 \rightarrow c^2 = 4$$

$$\frac{-9}{a^2} - 0 + \frac{16}{4} = 1 \rightarrow a^2 = 3$$

$$\frac{-36}{3} - \frac{16}{b^2} + \frac{64}{4} = 1$$

$$b^2 = \frac{16}{3}$$

So:

$$\boxed{-\frac{x^2}{3} - \frac{3y^2}{16} + \frac{z^2}{4} = 1}$$

5. **[7 points]** Consider the vector function $\mathbf{r}(t) = \langle t^2 - 3t, \sqrt{t+1}, t^2 + t \rangle$.

   Let ℓ be the line tangent to the space curve of $\mathbf{r}(t)$ at the point $(0, 2, 12)$.

   Write parametric equations for ℓ.

$\sqrt{t+1} = 2$

so $t = 3$

$$\vec{r}\,'(t) = \left\langle 2t - 3, \frac{1}{2\sqrt{t+1}}, 2t + 1 \right\rangle$$

$$\vec{r}\,'(3) = \left\langle 3, \frac{1}{4}, 7 \right\rangle \leftarrow \text{direction of line}$$

So:

$$\boxed{\begin{aligned} x &= 3t \\ y &= 2 + \tfrac{1}{4}t \\ z &= 12 + 7t \end{aligned}}$$

6. **[10 points]** The acceleration of a friendly bee at time t is given by the vector function

$$\mathbf{a} = \langle 6, 6t, e^t \rangle.$$

At time $t = 0$, the bee is at the origin. At time $t = 2$, the bee is at the point $(4, 5, 6)$. Where is the bee at time $t = 3$?

$$\vec{v}(t) = \left\langle 6t + C_1,\ 3t^2 + C_2,\ e^t + C_3 \right\rangle$$

$$\vec{r}(t) = \left\langle 3t^2 + C_1 t + C_4,\ t^3 + C_2 t + C_5,\ e^t + C_3 t + C_6 \right\rangle$$

$$\vec{r}(0) = \left\langle C_4, C_5, 1 + C_6 \right\rangle = \langle 0, 0, 0 \rangle \quad \text{so} \quad \begin{array}{l} C_4 = 0 \\ C_5 = 0 \\ C_6 = -1 \end{array}$$

$$\vec{r}(2) = \left\langle 12 + 2C_1,\ 8 + 2C_2,\ e^2 + 2C_3 - 1 \right\rangle = \langle 4, 5, 6 \rangle$$

$$\text{so} \quad \begin{array}{l} C_1 = -4 \\ C_2 = \frac{-3}{2} \\ C_3 = \frac{7 - e^2}{2} \end{array}$$

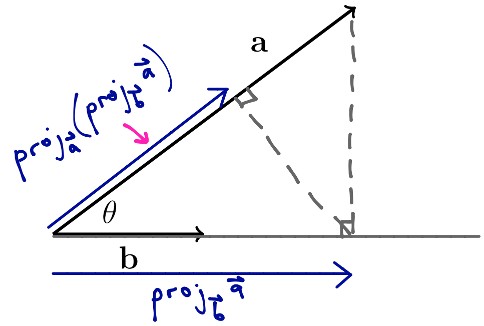
$$\text{so: } \vec{r}(t) = \left\langle 3t^2 - 4t,\ t^3 - \tfrac{3}{2}t,\ e^t + \left(\tfrac{7 - e^2}{2}\right)t - 1 \right\rangle$$

$$\vec{r}(3) = \left\langle 15,\ 22.5,\ e^3 + \left(\frac{7 - e^2}{2}\right)3 - 1 \right\rangle$$

so the bee is at $\boxed{\left(15, 22.5, e^3 - \tfrac{3}{2}e^2 + 9.5\right)}$

7. I've got some vectors **a** and **b**. Here's a picture:

(a) **[2 points]** In the picture to the right, draw $\mathbf{proj_a}(\mathbf{proj_b a})$. (Clearly label your work so I can see what's going on.)



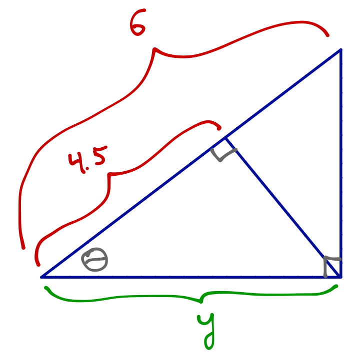
(b) **[6 points]** Suppose I know the following:

- $\mathbf{a} = \langle 4, 2, 4 \rangle$.
- $\mathbf{proj_a}(\mathbf{proj_b a}) = \langle 3, \frac{3}{2}, 3 \rangle$.
- θ is acute.

What's θ?

$$|\vec a| = \sqrt{16+4+16} = 6$$

$$\left|\text{proj}_{\vec a}(\text{proj}_{\vec b}\vec a)\right| = \sqrt{9 + \frac{9}{4} + 9} = 4.5$$



$$\cos\theta = \frac{4.5}{y}, \quad \text{and} \quad \cos\theta = \frac{y}{6}$$

so $$\cos\theta = \frac{4.5}{6\cos\theta}$$

$$\downarrow$$

$$\cos^2\theta = \frac{3}{4}$$

$$\downarrow$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \boxed{\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)} = \boxed{\frac{\pi}{6}\text{ rad}} \text{ or } \boxed{30^\circ}$$