Math 126 F - Winter 2018
Midterm Exam Number One
February 1, 2018

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1	12	
2	12	
3	12	
4	8	
5	8	
6	8	
Total	60	

- This exam consists of **six** problems on **four** double-sided pages.
- Show all work for full credit.
- You may use a TI-30X IIS calculator during this exam. Other calculators and electronic devices are not permitted.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- $\boxed{\text{Draw a box}}$ around your final answer to each problem.
- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.
- If you run out of room, write on the back of the first or last page and indicate that you have done so. If you still need more room, raise your hand and ask for an extra page.
- You may use one hand-written double-sided 8.5″ by 11″ page of notes.
- You have 80 minutes to complete the exam.

1. **[6 points per part]** Let $\mathcal{P}$ be the plane

$$2x + 3y - 5z = -9.$$

(a) Find the point on $\mathcal{P}$ that's closest to the point $(-9, -2, 16)$.

Line through $(-9,-2,16)$ orthogonal to P: $x = -9 + 2t$
$y = -2 + 3t$
$z = 16 - 5t$

Intersection of line w/ P: $2(-9+2t) + 3(-2+3t) - 5(16-5t) = -9$

$-18 + 4t - 6 + 9t - 80 + 25t = -9$

$38t = 95$

$t = 2.5$

$\boxed{(-4, 5.5, 3.5)}$

(b) Let ℓ be the line

$$x - 2 = -y = \frac{z-1}{3}.$$

Give parametric equations for the line within $\mathcal{P}$ that intersects ℓ at a right angle.

Where does ℓ intersect P? $x = 2 + t$
$y = -t$
$z = 1 + 3t$
$\rightarrow 2x + 3y - 5z = -9$

$2(2+t) + 3(-t) - 5(1+3t) = -9$

$4 + 2t - 3t - 5 - 15t = -9$

$-16t = -8$

$t = \frac{1}{2}$

$(2.5, -0.5, 2.5)$

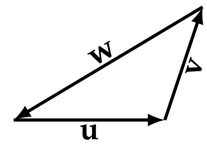
What direction is perp. to ℓ and in the plane?

$$\begin{array}{r} \langle 1, -1, 3 \rangle \\ \times \langle 2, 3, -5 \rangle \\ \hline \langle -4, 11, 5 \rangle \end{array}$$

$$\boxed{\begin{aligned} x &= 2.5 - 4t \\ y &= -0.5 + 11t \\ z &= 2.5 + 5t \end{aligned}}$$

2. **[3 points per part]**

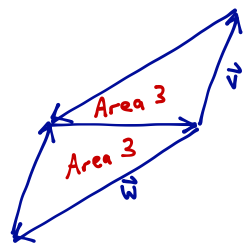
The 3-dimensional vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ point along the edges of a triangle, as shown below. The area of the triangle is 3, and $|\mathbf{u}| = 5$.



(a) Compute $|\mathbf{v} \times \mathbf{w}|$.

area of this parallelogram:

$\boxed{6}$



(b) Compute $|\mathbf{u} \times (\mathbf{v} \times \mathbf{w})|$.

$= |\vec{u}||\vec{v}\times w||\sin(\Theta)| = 5 \cdot 6 \cdot 1 = \boxed{30}$

angle btwn $\vec{u}$ & $\vec{v}\times\vec{w}$

$\vec{v}\times\vec{w}$ is orthogonal to the plane containing $\vec{u}, \vec{v}, \& \vec{w}$.

So $\Theta = \frac{\pi}{2}$, $\sin\Theta = 1$

(c) Compute $|(\mathbf{u} \times \mathbf{v}) \times (\mathbf{v} \times \mathbf{w})|$. $= \boxed{0}$

both are normal vectors to the plane of this triangle,

so $\vec{u}\times\vec{v}$ & $\vec{v}\times\vec{w}$ are parallel

(d) Suppose $\text{comp}_{\mathbf{v}}\mathbf{u} = 4$. What's $|\mathbf{v}|$?

$\text{Comp}_{\vec{v}}\vec{u} = |\vec{u}|\cos\Theta$

angle between $\vec{u}$ & $\vec{v}$

$4 = 5\cos\Theta$

$\cos\Theta = \frac{4}{5}$

$\sin\Theta = \sqrt{1-\cos^2\Theta} = \frac{3}{5}$

$|\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}||\sin\Theta|$

$6 = 5|\vec{v}|\left(\frac{3}{5}\right)$

$\boxed{|\vec{v}| = 2}$

3. **[4 points per part]** Suppose the position of a particle at time t is given by $\mathbf{r}(t) = \langle t, t^2, e^t \rangle$.

(a) Compute the tangential and normal components of acceleration at time $t = 0$.

$\vec{r}\,'(t) = \langle 1, 2t, e^t \rangle \qquad \vec{r}\,'(0) = \langle 1, 0, 1 \rangle$

$\vec{r}\,''(t) = \langle 0, 2, e^t \rangle \qquad \vec{r}\,''(0) = \langle 0, 2, 1 \rangle$

$$a_T = \frac{\vec{r}\,'(0) \cdot \vec{r}\,''(0)}{|\vec{r}\,'(0)|} = \frac{\langle 1,0,1 \rangle \cdot \langle 0,2,1 \rangle}{|\langle 1,0,1 \rangle|} = \boxed{\frac{1}{\sqrt{2}}}$$

$$a_N = \frac{|\vec{r}\,'(0) \times \vec{r}\,''(0)|}{|\vec{r}\,'(0)|} = \frac{|\langle 1,0,1 \rangle \times \langle 0,2,1 \rangle|}{|\langle 1,0,1 \rangle|} = \frac{|\langle -2,-1,2 \rangle|}{|\langle 1,0,1 \rangle|} = \boxed{\frac{3}{\sqrt{2}}}$$

(b) Compute the curvature at time $t = 0$.

$$K = \frac{|\vec{r}\,'(0) \times \vec{r}\,''(0)|}{|\vec{r}\,'(0)|^3} = \frac{|\langle -2,-1,2 \rangle|}{|\langle 1,0,1 \rangle|^3} = \boxed{\frac{3}{2\sqrt{2}}}$$

(c) Let P be the point where the particle intersects the plane $x = 2$. Find the acute angle between the velocity of the particle at P and a normal vector to the plane.

$x = t = 2$

$\vec{r}\,'(2) = \langle 1, 4, e^2 \rangle$

Normal vector $= \langle 1, 0, 0 \rangle$

$\langle 1, 4, e^2 \rangle \cdot \langle 1, 0, 0 \rangle = |\langle 1, 4, e^2 \rangle| \, |\langle 1, 0, 0 \rangle| \cos\theta$

$1 = \sqrt{17 + e^4} \cos\theta$

$$\theta = \boxed{\cos^{-1}\left(\frac{1}{\sqrt{17+e^4}}\right)} \text{ or } \boxed{1.452 \text{ rad}} \text{ or } \boxed{83.21^\circ}$$

4. Consider the following vector function:

$$\mathbf{r}(t) = \langle 2t\sin(t), \sqrt{t^2+1}, 3t\cos(t)\rangle$$

(a) **[6 points]** Give the equation for a quadric surface containing $\mathbf{r}(t)$.

Need a relationship btwn $x = 2t\sin(t)$, $y = \sqrt{t^2+1}$, $z = 3t\cos(t)$

$$\frac{x^2}{4} + \frac{z^2}{9} = t^2\sin^2(t) + t^2\cos^2(t) = t^2\left(\sin^2(t) + \cos^2(t)\right) = t^2$$

$$y^2 = t^2 + 1$$

So $\frac{x^2}{4} + \frac{z^2}{9} + 1 = y^2$

So: $\boxed{\frac{-x^2}{4} + y^2 - \frac{z^2}{9} = 1}$

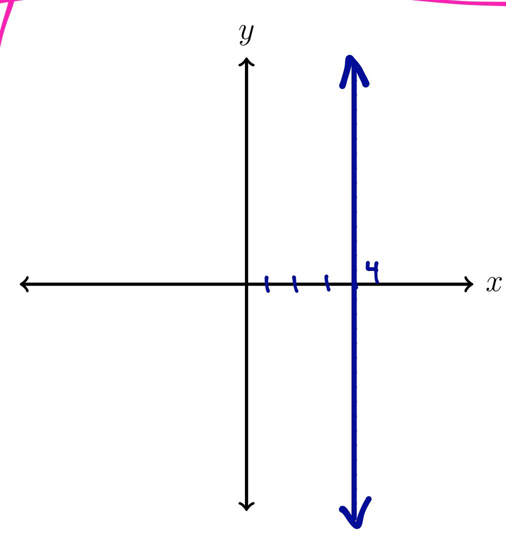
(b) **[2 points]** Write the name of the surface you found in part (a).

Hyperboloid of 2 sheets.

5. **[8 points]** Sketch the curve $r = 4\sec(\theta)$. Please label the scales on your axes.
(Don't *just* draw a picture. Show some reasoning.)

$x = r\cos\theta = 4\sec\theta\cos\theta = 4\left(\frac{1}{\cos\theta}\right)\cos\theta = 4$

So, $x = 4$:



6. **[8 points]** The force applied to Gomba after t seconds, in newtons, is

$$\mathbf{F} = 36t\mathbf{i} + 3\sin(t)\mathbf{j} + e^{\frac{t}{3}}\mathbf{k}.$$

Gomba has a mass of 9 kg and starts at rest at the origin.

Write a vector function for Gomba's position after t seconds.

$$\vec{a}(t) = \frac{\vec{F}}{m} = \left\langle 4t, \frac{1}{3}\sin(t), \frac{1}{9}e^{\frac{t}{3}} \right\rangle$$

$$\vec{v}(t) = \left\langle 2t^2 + C_1, \frac{-1}{3}\cos(t) + C_2, \frac{1}{3}e^{\frac{t}{3}} + C_3 \right\rangle$$

$$\vec{v}(0) = \left\langle C_1, \frac{-1}{3} + C_2, \frac{1}{3} + C_3 \right\rangle = \langle 0, 0, 0 \rangle$$

$$C_1 = 0 \quad C_2 = \frac{1}{3} \quad C_3 = \frac{-1}{3}$$

$$\vec{v}(t) = \left\langle 2t^2, \frac{-1}{3}\cos(t) + \frac{1}{3}, \frac{1}{3}e^{\frac{t}{3}} - \frac{1}{3} \right\rangle$$

$$\vec{r}(t) = \left\langle \frac{2}{3}t^3 + C_4, \frac{-1}{3}\sin(t) + \frac{t}{3} + C_5, e^{\frac{t}{3}} - \frac{t}{3} + C_6 \right\rangle$$

$$\vec{r}(0) = \langle C_4, C_5, 1 + C_6 \rangle = \langle 0, 0, 0 \rangle$$

$$C_4 = 0 \quad C_5 = 0 \quad C_6 = -1$$

$$\boxed{\vec{r}(t) = \left\langle \frac{2}{3}t^3, \frac{-1}{3}\sin(t) + \frac{t}{3}, e^{\frac{t}{3}} - \frac{t}{3} - 1 \right\rangle}$$