Math 126 A and B Midterm 1 Spring 2025

Print Your Full Name	Signature
Solutions	
Student ID Number	**Quiz Section**
Instructor's Name	**TA's Name**

Please read these instructions!

1. Your exam contains 6 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. You are allowed a single, double-sided 8.5"x11" handwritten notesheet; the TI-30XIIS calculator; a writing utensil; and an eraser to use on the exam.
3. The exam is worth 50 points. Point values for problems vary and these are clearly indicated. You have 80 minutes for this exam.
4. Unless otherwise stated, make sure to SHOW YOUR WORK CLEARLY. Credit is awarded to work which is clearly shown and legible. Full credit may not be awarded if work is unclear or illegible.
5. For problems that aren't sketches, place a box around your final answer to each question.
6. If you need extra space, use the last two pages of the exam. Clearly indicate on that there is more work located on the last pages, and indicate on those pages the related problem number.
7. Unless otherwise instructed, always give your answers in exact form. For example, 3π, $\sqrt{2}$, and $\ln(2)$ are in exact form; the corresponding approximations 9.424778, 1.4142, and 0.693147 are NOT in exact form.
8. Credit is awarded for correct use of techniques or methods discussed in class thus far. Partial credit may be awarded as earned. No credit is awarded for use of methods that are learned later in the course or from other courses.

Problem	Total Points
1	10
2	10
3	10
4	10
5	10
Total	50

1. (10 points) Find the area of the triangle with points $P(3,0,3), Q(-2,1,5)$ and $R(7,2,7)$. Do not simplify your final answer.

$\vec{PQ} = \langle -5, 1, 2 \rangle$ +2

$\vec{PR} = \langle 4, 2, 4 \rangle$ +2

+2 +2

area of triangle $= \frac{1}{2} \left| \vec{PQ} \times \vec{PR} \right| = \frac{1}{2} \left| \langle 0, 28, -14 \rangle \right|$

$= \boxed{\frac{1}{2}\sqrt{28^2 + 14^2}}$

+2

2. (10 points) Find an equation of the plane containing the line with parametrization $\mathbf{r}(t) = \langle 1+t, 3-4t, 5t \rangle$ and the point $P(1,2,3)$. Write your equation in the form $Ax + By + Cz = D$.

· P is not on line: $1 = 1 + t \Rightarrow t = 0$

but $2 \neq 3 - 4(0)$, $3 \neq 5(0)$. +2

· Point on line: $(1, 3, 0)$, Direction vector: $\vec{v}_1 = \langle 1, -4, 5 \rangle$

+2

Second vector on plane: $\vec{v}_2 = \langle 1-1, 2-3, 3-0 \rangle$

$= \langle 0, -1, 3 \rangle$ +2

· $\vec{n} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -4 & 5 \\ 0 & -1 & 3 \end{vmatrix} = \langle -12+5, 0-3, -1 \rangle$

$= \langle -7, -3, -1 \rangle$ +2

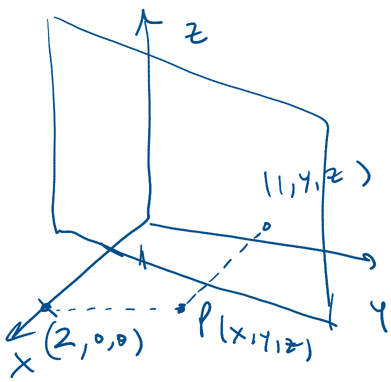
$\Rightarrow -7(x-1) - 3(y-3) - z = 0$

$-7x + 7 - 3y + 9 - z = 0$

$\boxed{-7x - 3y - z = -16}$

+2

3. (10 points) Find an equation for the surface consisting of all points $P(x, y, z)$ for which the distance from P to the point $(2, 0, 0)$ is $\sqrt{2}$ times the distance from P to the plane $x = 1$. Then identify the surface by name.



+3 +3

$$\sqrt{(x-2)^2+(y-0)^2+(z-0)^2} = \sqrt{2}\cdot\sqrt{(x-1)^2+0^2+0^2}$$

$$(x-2)^2+y^2+z^2 = 2\cdot(x-1)^2$$

$$x^2-4x+4+y^2+z^2 = 2(x^2-2x+1)$$

$$x^2\cancel{-4x}+4+y^2+z^2 = 2x^2\cancel{-4x}+2$$

$$\boxed{-x^2+y^2+z^2=-2}$$ +3

hyperboloid of two sheet +1

4. (10 points) A particle moves with acceleration function $\mathbf{a}(t) = \langle \cos t, \sin t, t\rangle$ and initial velocity $\mathbf{v}(0) = \langle 0, -1, 0\rangle$. Find the tangential component of acceleration as a function of t.

$$\vec{v}(t) = \left\langle \sin t + C_1, -\cos t + C_2, \frac{t^2}{2}+C_3 \right\rangle$$

$$\vec{v}(0) = \langle C_1, -1+C_2, C_3\rangle = \langle 0,-1,0\rangle \rightarrow \begin{matrix} C_1 = 0 \\ C_2 = 0 \\ C_3 = 0 \end{matrix}$$

$$\vec{v}(t) = \left\langle \sin t, -\cos t, \frac{t^2}{2}\right\rangle$$ +4

$$a_T = \frac{\vec{a}\cdot\vec{v}}{|\vec{v}|} = \frac{\langle \cos t, \sin t, t\rangle \cdot \langle \sin t, -\cos t, \frac{t^2}{2}\rangle}{\sqrt{\sin^2 t + \cos^2 t + \frac{t^4}{4}}}$$

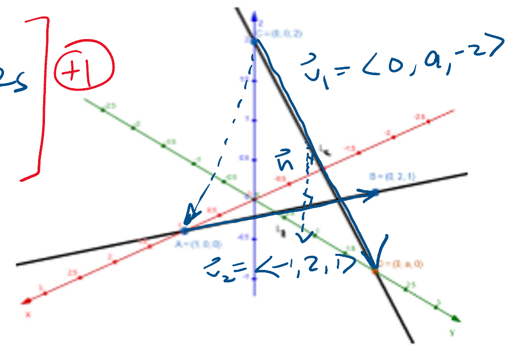
$$= \boxed{\frac{\frac{t^3}{2}}{\sqrt{1+\frac{t^4}{4}}}}$$ +3 +3

5. Let L_1 be the line through points $A(1,0,0)$ and $B(0,2,1)$. Let L_2 be the line through points $C(0,0,2)$ and $D(0,a,0)$, where a is a variable. (See the figure below.)

(a) (6 points) Find the value of a such that lines L_1 and L_2 intersect.

L_1: $x = 0 + 0t = 0$, $y = 0 + at = at$, $z = 2 - 2t$ (+2)

L_2: $x = 1 - s$, $y = 0 + 2s = 2s$, $z = 0 + s = s$ (+1)



x: $0 = 1 - s$, $1 = s$ (+1)

z: $2 - 2t = s$, $2 - 2t = 1$, $-2t = -1$, $t = \frac{1}{2}$ (+1)

y: $at = 2s$, $a \cdot \frac{1}{2} = 2$, $\boxed{a = 4}$ (+1)

(b) (4 points) Now assume a is not the value found in part (a), so that the lines are skew (you do not need to verify this). Find the values of a such that the distance between the lines is 1. [Hint: Find a vector between L_1 and L_2. Then calculate a scalar component.]

$$\vec{n} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & a & -2 \\ -1 & 2 & 1 \end{vmatrix} = \langle a+4, 2, a \rangle \quad (+1)$$

$$\vec{CA} = \langle 1, 0, -2 \rangle \quad (+1)$$

$$1 = |\text{comp}_{\vec{n}} \vec{CA}| = \frac{|\vec{CA} \cdot \vec{n}|}{|\vec{n}|} = \frac{|a + 4 - 2a|}{\sqrt{(a+4)^2 + 4 + a^2}}$$

$$1 = \frac{|4 - a|}{\sqrt{a^2 + 8a + 16 + 4 + a^2}} \quad (+1)$$

$$\sqrt{2a^2 + 8a + 20} = |4 - a|$$

$$2a^2 + 8a + 20 = (4 - a)^2$$

$$2a^2 + 8a + 20 = 16 - 8a + a^2$$

$$a^2 + 16a + 4 = 0$$

$$a = \frac{-16 \pm \sqrt{16^2 - 16}}{2} = \frac{-16 \pm 4\sqrt{15}}{2}$$

$$\boxed{a = -8 \pm 2\sqrt{15}} \quad (+1)$$

Extra scratch paper.

Extra scratch paper.