1. **[5 points per part]**

(a) Write an equation for the plane through the points $(1, 6, 0)$, $(2, 4, -1)$, and $(1, 2, 4)$.

A B C

$\overrightarrow{AB} = \langle 1, -2, -1 \rangle$

$\overrightarrow{AC} = \langle 0, -4, 4 \rangle$

$\overrightarrow{AB} \times \overrightarrow{AC} = \langle -12, -4, -4 \rangle$

Normal vector: $\langle 3, 1, 1 \rangle$

(or any vector parallel to this)

$3(x-1) + 1(y-6) + 1(z-0) = 0$

$\boxed{3x + y + z = 9}$

(b) Find the (acute) angle between the plane from part (a) and the plane $4x - y + 3z = 0$.

= angle btwn normal vectors: $\vec{a} = \langle 3, 1, 1 \rangle$

$\vec{b} = \langle 4, -1, 3 \rangle$

$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$

$14 = \sqrt{11}\sqrt{26}\cos\theta$

$\theta = \boxed{\cos^{-1}\left(\frac{14}{\sqrt{11}\sqrt{26}}\right)} \approx \boxed{0.59555}$ or $\boxed{34.1228^\circ}$

(c) Write parametric equations for the intersection of the two planes from (a) and (b).

$$\begin{array}{r} \langle 3, 1, 1 \rangle \\ \times \langle 4, -1, 3 \rangle \\ \hline \end{array}$$

Direction of intersection: $\langle 4, -5, -7 \rangle$

$$\boxed{\begin{aligned} x &= 4t \\ y &= \tfrac{27}{4} - 5t \\ z &= \tfrac{9}{4} - 7t \end{aligned}}$$

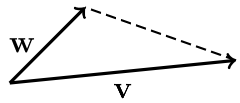
Some point in intersection: $3x + y + z = 9$

$4x - y + 3z = 0$ → $\left(0, \frac{27}{4}, \frac{9}{4}\right)$

arbitrary → $x = 0$

2. **[3 points per part]** Suppose $\mathbf{v} \times \mathbf{w} = \langle 2, -3, 6 \rangle$.

(a) Find the area of this triangle:



$= \frac{1}{2} |\vec{v} \times \vec{w}| = \frac{1}{2} \sqrt{2^2 + 3^2 + 6^2} = \boxed{3.5}$

(b) Compute $\mathbf{w} \times \mathbf{v}$.

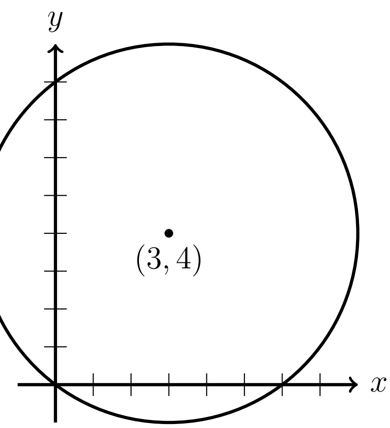
$= -(\vec{v} \times \vec{w}) = \boxed{\langle -2, 3, -6 \rangle}$

(c) Compute $(\mathbf{v} + \mathbf{v}) \times (\mathbf{v} + \mathbf{w})$.

$= \vec{v} \times \vec{v} + \vec{v} \times \vec{v} + \vec{v} \times \vec{w} + \vec{v} \times \vec{w}$

$= \vec{0} + \vec{0} + 2(\vec{v} \times \vec{w}) = \boxed{\langle 4, -6, 12 \rangle}$

3. **[7 points]** Write a polar equation for the following circle:



Your answer should be in the form $r = f(\theta)$.

$(x-3)^2+(y-4)^2=5^2$

$x^2-6x+9+y^2-8y+16=25$

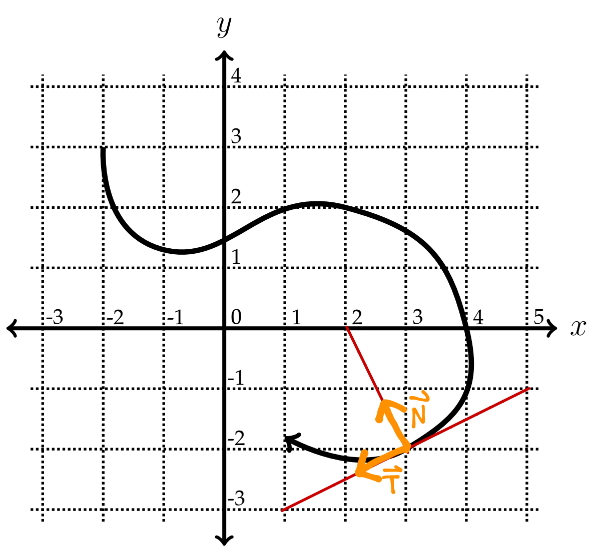
$x^2+y^2=6x+8y$

$x^2+y^2=r^2$
$x=r\cos\theta$
$y=r\sin\theta$

$r^2=6r\cos\theta+8r\sin\theta$

$\boxed{r=6\cos\theta+8\sin\theta}$

4. **[7 points]** Here's a drawing of the space curve of $\mathbf{r}(t) = \langle x(t), y(t), 0\rangle$ in the xy-plane.



Find $\mathbf{T}$ and $\mathbf{N}$ at the point $(3, -2, 0)$.

$\vec{T}$ is a unit vector in the direction the curve is pointing:

$\vec{T} \approx \frac{1}{\sqrt{5}}\langle -2,-1,0\rangle = \boxed{\left\langle \frac{-2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}, 0\right\rangle}$

$\vec{N}$ is perpendicular to $\vec{T}$, in the direction the curve is turning:

$\vec{N} = \boxed{\left\langle \frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0\right\rangle}$

5. Consider the quadric surface $x^2 + 2y + z^2 + 2z = 4$.

(a) **[6 points]** Write the name for this surface.

(No credit for just writing an answer. Show your work.)

$$x^2 + 2y + z^2 + 2z + 1 = 5$$

$$x^2 + 2y + (z+1)^2 = 5$$

$$x^2 + (2y-5) + (z+1)^2 = 0$$

This is a $\boxed{\text{circular paraboloid}}$ (or elliptic paraboloid)

(b) **[9 points]** A particle travels along the path $\mathbf{r}(t) = \langle \sin(\pi t), 4+t, \cos(\pi t) - 1 \rangle$.

Find the particle's tangential and normal components of acceleration at the time when it hits the surface.

$$(\sin(\pi t))^2 + 2(4+t) - 5 + (\cos(\pi t) - 1 + 1)^2 = 0$$

$$\sin^2(\pi t) + \cos^2(\pi t) + 3 + 2t = 0$$

$$4 + 2t = 0$$

$$t = -2$$

$$\vec{r}\,'(t) = \langle \pi\cos(\pi t), 1, -\pi\sin(\pi t) \rangle \qquad \vec{r}\,'(-2) = \langle \pi, 1, 0 \rangle$$

$$\vec{r}\,''(t) = \langle -\pi^2\sin(\pi t), 0, -\pi^2\cos(\pi t) \rangle \qquad \vec{r}\,''(-2) = \langle 0, 0, -\pi^2 \rangle$$

$$a_T = \frac{\vec{r}\,'(-2)\cdot\vec{r}\,''(-2)}{|\vec{r}\,'(-2)|} = \frac{\langle \pi, 1, 0\rangle \cdot \langle 0, 0, -\pi^2\rangle}{\sqrt{1+\pi^2}} = \boxed{0}$$

$$a_N = \frac{|\vec{r}\,'(-2)\times\vec{r}\,''(-2)|}{|\vec{r}\,'(-2)|} = \frac{|\langle \pi, 1, 0\rangle \times \langle 0, 0, -\pi^2\rangle|}{\sqrt{1+\pi^2}} = \frac{|\langle -\pi^2, -\pi^3, 0\rangle|}{\sqrt{1+\pi^2}} = \frac{\sqrt{\pi^4+\pi^6}}{\sqrt{1+\pi^2}} = \boxed{\pi^2}$$

6. **[7 points]** Are the following lines parallel, intersecting, or skew?

$$\frac{x-9}{2} = \frac{y-5}{3} = 3 - z$$

$$\frac{z-3}{-1}$$

$x = 9 + 2t$
$y = 5 + 3t$
$z = 3 - t$

$$x = -8 + 8t$$
$$y = 5 - 5t$$
$$z = 1 + 3t$$

$9 + 2t = -8 + 8s$
$5 + 3t = 5 - 5s$
$3 - t = 1 + 3s \rightarrow t = 2 - 3s$

$5 + 3(2 - 3s) = 5 - 5s$
$5 + 6 - 9s = 5 - 5s$
$6 = 4s$
$s = 1.5$
$t = 2 - 4.5 = -2.5$

check?

$$9 + 2(-2.5) \stackrel{?}{=} -8 + 8(1.5)$$

$$4 = 4$$

Yup!

So, they $\boxed{\text{intersect.}}$