Solutions to Math 126 A MT1, Spring '18

1. Consider the following vector function

$$\mathbf{r}(t) = \langle \sin(t) - t\cos(t), \cos(t) + t\sin(t) \rangle, \quad t > 0.$$

(a) (8 points) Compute the tangential and normal components of acceleration.

$$\vec{r}\,'(t) = \langle \cos t - \cos t + t\sin t, -\sin t + \sin t + t\cos t \rangle$$
$$= t\langle \sin t, \cos t \rangle$$

$$|\vec{r}\,'(t)| = t$$

$$\boxed{a_T = |\vec{r}\,'(t)|' = 1}$$

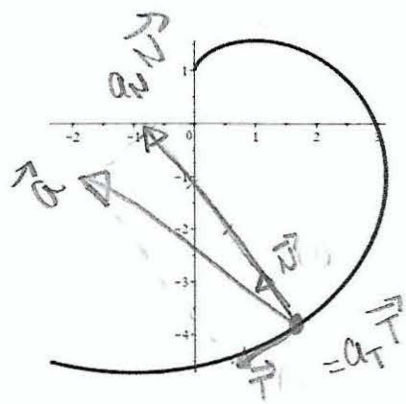
$$\vec{r}\,''(t) = \langle \sin t, \cos t \rangle + t\langle \cos t, -\sin t \rangle$$

$$\vec{r}\,' \times \vec{r}\,'' = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ t\sin t & t\cos t & 0 \\ \sin t + t\cos t & \cos t - t\sin t & 0 \end{vmatrix} = (t\sin t\cos t - t^2\sin^2 t - t\sin t\cos t - t^2\cos^2 t)\vec{k} = \langle 0, 0, -t^2 \rangle$$

$$|\vec{r}\,' \times \vec{r}\,''| = t^2$$

$$\boxed{a_N = \frac{t^2}{t} = t}$$

(b) (4 points) On the right is the graph of part of the curve $0 \le t \le 5$. Show $\mathbf{T}(4)$, $\mathbf{N}(4)$ and $\mathbf{a}(4)$ on the graph.

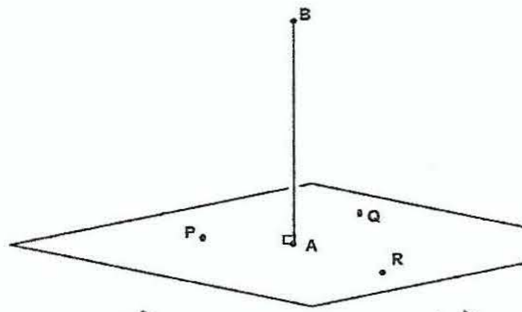


$$\vec{r}(4) = \langle \sin 4 - 4\cos 4, \cos 4 + 4\sin 4 \rangle \approx \langle 1.9, -3.7 \rangle$$

$$\vec{a}(4) = 1 \cdot \vec{T} + 4 \cdot \vec{N}$$

2. The line though the point $B(-2,3,1)$ perpendicular to the plane containing the points $P(1,2,3)$, $Q(0,-1,2)$ and $R(0,3,-1)$ intersects it at the point A as shown in the picture.

(a) (6 points) Find the equation of the plane and simplify.



$$\vec{n} = \vec{PQ} \times \vec{PR} = \langle -1,-3,-1\rangle \times \langle -1,1,-4\rangle$$

$$= \langle 1,3,1\rangle \times \langle 1,-1,4\rangle$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 1 \\ 1 & -1 & 4 \end{vmatrix}$$

$$= (12+1)\vec{i} - (4-1)\vec{j} + (-1-3)\vec{k} = \langle 13,-3,-4\rangle$$

$$13(x-1) - 3(y-2) - 4(z-3) = 0$$

$$13x - 3y - 4z = 13 - 6 - 12$$

$$\boxed{13x - 3y - 4z = -5}$$

(b) (4 points) Find the coordinates of the point A.

$A(x,y,z)$ $\quad \vec{AB} = \text{proj}_{\vec{n}} \vec{PB} = \dfrac{\langle 13,-3,-4\rangle \cdot \langle -3,1,-2\rangle}{\langle 13,-3,-4\rangle \cdot \langle 13,-3,-4\rangle} \langle 13,-3,-4\rangle$

$$\langle -2-x, 3-y, 1-z\rangle = \frac{-39-3+8}{169+9+16}\langle 13,-3,-4\rangle = \frac{-34}{194}\langle 13,-3,-4\rangle = \left\langle \frac{-221}{97}, \frac{51}{97}, \frac{68}{97}\right\rangle$$

$$x = -2 + \frac{221}{97} = \frac{27}{97} \qquad y = 3 - \frac{51}{97} = \frac{240}{97} \qquad z = 1 - \frac{68}{97} = \frac{29}{97}$$

$$A\left(\frac{27}{97}, \frac{240}{97}, \frac{29}{97}\right)$$

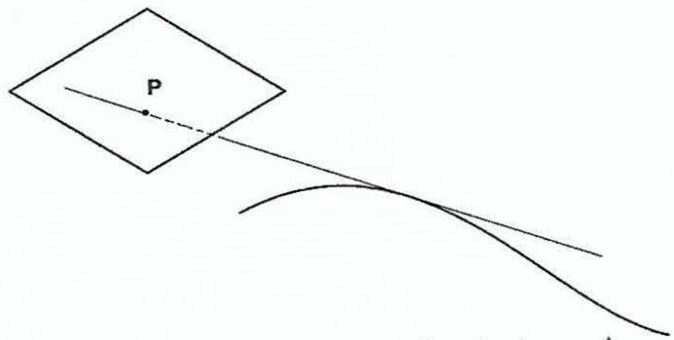
(c) (2 points) Find the distance from the plane to the point B.

$$|\vec{AB}| = \frac{34}{194}\sqrt{169+9+16} = \frac{34}{\sqrt{194}}$$

3. (10 points) The tangent line through the point with $t = 3$ to the curve

$$\mathbf{r}(t) = \langle t^2 + 1, 2t + 7, t^2 - t + 1 \rangle$$

intersects the plane $2x + 3y - 5z = 10$ at the point P as shown in the picture. Find the coordinates of the point P.



Tangent line:

$\vec{r}\,'(t) = \langle 2t, 2, 2t-1 \rangle$

$\vec{r}\,'(3) = \langle 6, 2, 5 \rangle$

$\vec{r}(3) = \langle 10, 13, 7 \rangle$

$\vec{r}_\ell(t) = \langle 10+6t, 13+2t, 7+5t \rangle$

Intersection

$2(10+6t) + 3(13+2t) - 5(7+5t) = 10$

$20 + 12t + 39 + 6t - 35 - 25t = 10$

$14 = 7t$

$\boxed{2 = t}$

point P $(10+6(2), 13+2(2), 7+5(2)) = (22, 17, 17)$

4. (6 points) The following curves are graphed with surfaces they are on.

$x^2+y^2=\sin^2 t$
$x^2+y^2+z^2=1$

A. $\mathbf{r}(t) = \langle \sin(t)\cos(14t), \sin(t)\sin(14t), \cos(t)\rangle$ **B.** $\mathbf{r}(t) = \langle t\cos(2t), t\sin(2t), t+1\rangle$ $x^2+y^2=t^2=(z-1)^2$

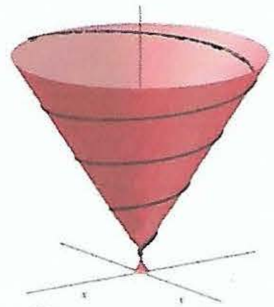
C. $\mathbf{r}(t) = \langle 2\cos(t), 3\sin(t), 3-4\cos(t)-9\sin(t)\rangle$ **D.** $\mathbf{r}(t) = \langle 2\cos(5t), 3\sin(5t), t\rangle$ $\left(\frac{x}{2}\right)^2+\left(\frac{y}{3}\right)^2=1$

$z+2x+3y=3$

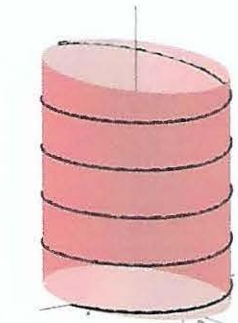
$x^2+z^2=1$? **E.** $\mathbf{r}(t) = \langle \cos(3t), t, \sin(3t)\rangle$ **F.** $\mathbf{r}(t) = \langle t, t\sin(t), (t-1)^2+3\rangle$ $z=(x-1)^2+3$

$z=\sin(3y)$

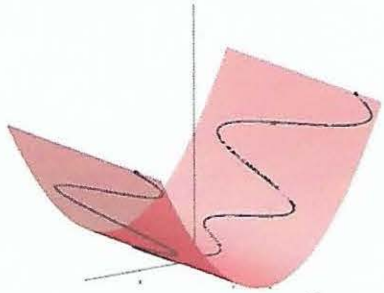
Under each picture, write down the letter of the curve and the equation of the surface it is on. For example: $\mathbf{G}, x+y^2+z^3=1$. Use the curve equations to get the surface equations. You have to get both right to get the point for each part. The positive z-axis points up in all the pictures.



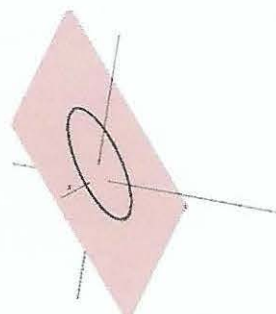
B, $x^2+y^2=(z-1)^2$



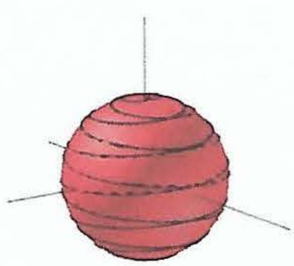
D, $\left(\frac{x}{2}\right)^2+\left(\frac{y}{3}\right)^2=1$



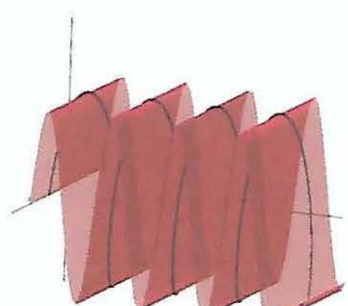
F, $z=(x-1)^2+3$



C, $2x+3y+z=3$



A, $x^2+y^2+z^2=1$



E, $z=\sin(3y)$