MATH 126 C/D MIDTERM 1 Spring 2016

April 21, 2016

NAME: SIGNATURE:

STUDENT ID #: TA SECTION:

Problem	Number of points	Points obtained
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Instructions:

- Your exam consists of FIVE problems. Please check that you have all of them.
- No books or notebooks allowed; you may use an A4 double-sided, handwritten sheet of notes *for personal use* (do not share).
- Place a box around your final answer to each question.
- The only calculator allowed is the **TI 30X IIS** of any color. All other electronic devices are prohibited.
- **Answers with little or no justification may receive no credit.**
- **Answers obtained by guess-and-check work will receive little or no credit, even if correct.**
- Read problems *carefully.*
- Raise your hand if you have a question.
- If you need more space, use additional blank sheets which will be provided by your TA. It is your responsibility to have him/her staple the additional sheets to your exam before you turn it in.
- Please turn off cell phones. GOOD LUCK!

Problem 1. (10 pts) Consider the planes $3x - y + 3z = 10$ and $4y - z = 2$. Give a parametric representation of the line of intersection of these two planes.

Solution. The direction vector v of the line is perpendicular to both the normal vectors $n_1 = \langle 3, -1, 3\rangle$ and $n_2 = \langle 0, 4, -1\rangle$. Thus, we can take $v = n_1 \times n_2 = \langle -11, 3, 12\rangle$. We now find one point that lies in the intersection by taking $z = 0$. We get $(7/2, 1/2, 0)$ lies in the intersection. Thus, one possible parametrization of the line is

$$\langle 7/2 - 11t, 1/2 + 3t, 12t\rangle.$$

Problem 2. (10 pts) Answer the following multiple choice questions. You need not show any work.

(a) Let $\mathbf{v}$ and $\mathbf{w}$ be two non-zero vectors in $\mathbf{R}^3$ such that $|\mathbf{v} \times \mathbf{w}| = 0$. Which of these statements must be true? Give all correct responses.

(i) $|\mathbf{v} \cdot \mathbf{w}| = 0$.

(ii) $|\text{Proj}_{\mathbf{v}}\mathbf{w}| = |\mathbf{w}|$

(iii) $|\text{Proj}_{\mathbf{v}}\mathbf{w}| = |\mathbf{v}|$

(iv) If $\mathbf{u}$ is another vector such that $\mathbf{u} \cdot \mathbf{v} = 0$, then $\mathbf{u} \cdot \mathbf{w}$ must also be zero.

(ii) and (iv)

(b) Let $\mathbf{v}$ and $\mathbf{w}$ be two non-zero vectors in $\mathbf{R}^3$. Which of these statements must be true? Give all correct responses.

(i) $|\text{Proj}_{\mathbf{v}}\mathbf{w}| \leq |\mathbf{w}|$

(ii) $|\text{Proj}_{\mathbf{v}}\mathbf{w}| \leq |\mathbf{v}|$

(iii) $|\text{Proj}_{\mathbf{v}}\mathbf{w}| \leq |\mathbf{w} \cdot \mathbf{v}|$

(iv) $|\text{Proj}_{\mathbf{v}}\mathbf{w}| \geq |\mathbf{w} \cdot \mathbf{v}|$

(i)

(c) Which of the following statements must be true.

(i) If we have two lines in $\mathbf{R}^3$ which intersect at a point, then there is a plane that contains them both.

(ii) If we have a one line and a point not on that line in $\mathbf{R}^3$, then there is a plane that contains them both.

(i) and (ii)

Problem 3. (10 pts) Find the distance of the origin in $\mathbf{R}^3$ from the line

$$x = 1 + t, \quad y = 2 - t, \quad z = -1 + 2t.$$

Hint: Find a point on the line such that the vector starting at the origin and ending at that point is perpendicular to the line. Distance of the origin is the magnitude of that vector.

Solution. We find t such that $(1 + t, 2 - t, -1 + 2t) \cdot (1, -1, 2) = 0$. This gives us

$$0 = 1 + t - 2 + t - 2 + 4t = -3 + 6t.$$

Thus $t = 1/2$ and the point is $(3/2, 3/2, 0)$. Hence, distance of origin is $3/\sqrt{2}$.

Problem 4. (10 pts) Two particles travel along the space curves

$$\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle, \quad \mathbf{r}_2(t) = \langle 1 + 6t, 1 + 30t, 1 + 126t \rangle.$$

Find the points at which their paths intersect. Do these particles ever collide?

Solution. We look for t, s such that $r_1(t) = r_2(s)$. Thus

$$t = 1 + 6s,\ t^2 = 1 + 30s,\ t^3 = 1 + 126s.$$

Comparing the first two equations we get $t^2 = 1 + 5(t-1) = 5t - 4$. Thus $t^2 - 5t + 4 = 0$. This gives us $(t, s) = \{(1, 0), (4, 1/2)\}$. Both solutions solve all three equations. Hence, there are both valid. Thus, points at which their paths intersect are $\langle 1, 1, 1 \rangle$ and $\langle 4, 16, 64 \rangle$. They never collide since $t \neq s$ for both solutions.

Problem 5. (10 pts) Consider the curve $x = 3\cos t$, $y = \cos 2t$ for $0 \leq t \leq 2\pi$. Find all points (x, y) where the tangent to the curve is horizontal.

Solution. $x'(t) = -3\sin(t)$, $y'(t) = -2\sin(2t)$. Hence $dy/dx = 2\sin(2t)/(3\sin(t)) = 2/3\cos(t)$. This is zero when $\cos(t) = 0$ or $t = \pi/2, 3\pi/2$. Thus the points of horizontal tangent are

$$(0, -1) \quad \text{and} \quad (0, 1).$$

Extra sheet.