Your Name

Your Signature

Student ID #

Your TA's name

Your Quiz Section Label and Time

Problem	Points	Possible
1		10
2		7
3		19
4		14
Total		50

- No books allowed.
- You may use a scientific calculator and one $8\frac{1}{2} \times 11$ sheet of notes.
- Do not share notes.
- In order to receive credit, you must show your work and explain your reasoning (except on the "short answer" questions).
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the grader where to find your work.
- Raise your hand if you have a question or need more paper.

Don't open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!

1. **(10 points)** Let **a**, **b** and **c** be three nonzero coplanar vectors (that is, they lie in the same plane) in $\mathbf{R}^3$, and assume that no two of them are parallel. Let $\mathbf{v} = \mathbf{a} \times (\mathbf{b} \times \mathbf{c})$. For each of the following statements determine whether it is True (**T**) or False (**F**).
No explanation of answers is needed for this problem. Be sure to explain your answers on other problems!

   (a) **v** is the zero vector. **T** **F**

   (b) $\mathbf{v} = (\mathbf{b} \times \mathbf{c}) \times \mathbf{a}$ **T** **F**

   (c) $\mathbf{v} \cdot (\mathbf{b} \times \mathbf{c}) = 0$. **T** **F**

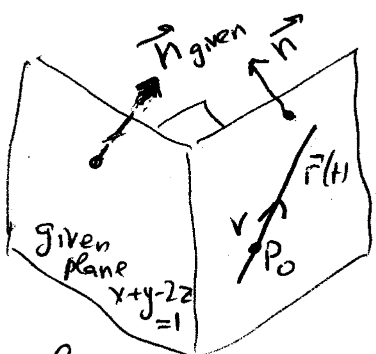
   (d) **v** is perpendicular to the plane containing vectors **a**, **b** and **c**. **T** **F**

   (e) **v** is parallel to the plane containing vectors **a**, **b** and **c**. **T** **F**

2. **(7 points)** Write an equation of the plane that contains the line $\mathbf{r}(t) = \langle -2+t, 3-2t, t\rangle$ and is perpendicular to the plane $x + y - 2z = 1$.

• need a point: take any point on the line $\vec{r}(t)$
e.g. $\boxed{P_0(-2,3,0)}$

• need a normal vector:
we have $\vec{v} = \langle 1, -2, 1\rangle$
also $\vec{n}_{\text{given plane}} = \langle 1, 1, -2\rangle$



The normal to the plane in question should be $\perp$ to $\vec{v}$ (Because $\vec{v}$ // to this plane) and also $\perp$ to $\vec{n}_{given}$ (Because the two planes are perpendicular)

$\Longrightarrow$ we can take

$\vec{n}$ to be a multiple of $\vec{v} \times \vec{n}_{given} = \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{vmatrix}$

$= \langle 3, 3, 3\rangle = 3\boxed{\langle 1, 1, 1\rangle}$

$\Longrightarrow$ one possible equation is $1(x-(-2)) + 1(y-3) + 1(z-0) = 0$

or $\boxed{x + y + z = 1}$

3. (19 = 2 + 5 + 7 + 5 **points**) Consider the curve $\mathbf{r}(t) = \langle -e^t, e^t \sin t, e^t \cos t \rangle$.

(a) Show that this curve lies on the cone $x^2 = y^2 + z^2$.

$y^2+z^2 = (e^t \sin t)^2 + (e^t \cos t)^2 = e^{2t}(\underbrace{\sin^2 t + \cos^2 t}_{1}) = e^{2t} = (-e^t)^2 = x^2$ ✓

(b) Find parametric equations for the tangent line to this curve at the point $(-1, 0, 1)$.

$(-1,0,1) = (-e^t, e^t \sin t, e^t \cos t) \Rightarrow \boxed{t=0}$

$\vec{r}'(t) = \langle -e^t, e^t(\sin t + \cos t), e^t(\cos t - \sin t)\rangle \Rightarrow$

$\vec{V} = \vec{r}'(0) = \boxed{\langle -1, 1, 1\rangle}$

$\Rightarrow$ parametric eqns for this line are $\boxed{\begin{cases} x = -1 - t \\ y = t \\ z = 1 + t \end{cases}}$

(c) Find the curvature of this curve at the point $(-1, 0, 1)$.

$\vec{r}'(t) = \langle -e^t, e^t(\sin t + \cos t), e^t(\cos t - \sin t)\rangle$

$\vec{r}''(t) = \langle -e^t, e^t(\sin t + 2\cos t - \sin t), e^t(\cos t - 2\sin t - \cos t)\rangle$

$= \langle -e^t, 2e^t \cos t, -2e^t \sin t\rangle$

$\Rightarrow \begin{cases} \vec{r}'(0) = \langle -1, 1, 1\rangle \rightsquigarrow |\vec{r}'(0)| = \sqrt{3} \\ \vec{r}''(0) = \langle -1, 2, 0\rangle \end{cases}$

$\Rightarrow \vec{r}'(0) \times \vec{r}''(0) = \langle -2, -1, -1\rangle \Rightarrow |\vec{r}'(0) \times \vec{r}''(0)| = \sqrt{4+1+1} = \sqrt{6}$

$\Rightarrow K(0) = \frac{\sqrt{6}}{(\sqrt{3})^3} = \frac{\sqrt{6}}{3\sqrt{3}} = \boxed{\frac{\sqrt{2}}{3}}$

(d) Find the length of the portion of this curve between the points $(-1, 0, 1)$ and $(-e^{\pi/2}, e^{\pi/2}, 0)$.

$(-e^{\pi/2}, e^{\pi/2}, 0) \rightsquigarrow t = \pi/2$

$(-1, 0, 1) \rightsquigarrow t = 0$

$\Rightarrow L = \int_0^{\pi/2} |\vec{r}'(t)|\,dt = \int_0^{\pi/2} \sqrt{e^{2t} + e^{2t}(\sin t + \cos t)^2 + e^{2t}(\cos t - \sin t)^2}\,dt$

$= \int_0^{\pi/2} e^t \sqrt{1 + (\sin^2 t + \cancel{2\sin t \cdot \cos t} + \cos^2 t) + (\cos^2 t - \cancel{2\sin t \cdot \cos t} + \sin^2 t)}\,dt$

$= \int_0^{\pi/2} e^t \sqrt{1 + 2\sin^2 t + 2\cos^2 t}\,dt = \int_0^{\pi/2} e^t \cdot \sqrt{3}\,dt = \sqrt{3}e^t \Big|_0^{\pi/2}$

$= \boxed{\sqrt{3}(e^{\pi/2} - 1)}$

4. (14 = 6+4+4 **points**) Consider the following two curves: one is represented by the Cartesian equation $x + y = 2$, and another one by the polar equation $r = \cos\theta - \sin\theta$.

(a) Find the slope of the tangent line to the second curve at the point corresponding to $\theta = \pi/4$.

$$\begin{cases} x = r\cos\theta = \cos^2\theta - \cos\theta\sin\theta = \cos^2\theta - \frac{1}{2}\sin(2\theta) \\ y = r\sin\theta = \cos\theta\sin\theta - \sin^2\theta = \frac{1}{2}\sin(2\theta) - \sin^2\theta \end{cases}$$

$$\Rightarrow \frac{dx}{d\theta} = -2\cos\theta\sin\theta - \frac{1}{2}\cdot 2\cdot\cos(2\theta) = -\sin(2\theta) - \cos(2\theta)$$

$$\Rightarrow \left.\frac{dx}{d\theta}\right|_{\theta=\frac{\pi}{4}} = -1 - 0 = -1$$

$$\frac{dy}{d\theta} = \frac{1}{2}\cdot 2\cdot\cos(2\theta) - 2\sin\theta\cdot\cos\theta = \cos(2\theta) - \sin(2\theta)$$

$$\Rightarrow \left.\frac{dy}{d\theta}\right|_{\theta=\frac{\pi}{4}} = 0 - 1 = -1$$

$$\Rightarrow \text{slope} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=\frac{\pi}{4}} = \frac{-1}{-1} = \boxed{1}$$

(b) Find a polar equation for the first curve.

$$\left.\begin{array}{l} x+y=2 \\ x = r\cos\theta \\ y = r\sin\theta \end{array}\right\} \Rightarrow r(\cos\theta + \sin\theta) = 2 \Rightarrow \boxed{r = \frac{2}{\cos\theta + \sin\theta}}$$

(c) Find the points of intersection of these two curves, if any. Show your work!

Soln 1 We are intersecting

$$\left.\begin{array}{l} r = \frac{2}{\cos\theta + \sin\theta} \text{ with} \\ r = \cos\theta - \sin\theta \end{array}\right\} \Rightarrow \frac{2}{\cos\theta + \sin\theta} = \cos\theta - \sin\theta$$

$$\Rightarrow 2 = \cos^2\theta - \sin^2\theta$$

$$\Rightarrow 2 = \cos(2\theta)$$

This eqn has no solutions,

so $\boxed{\text{there are no points of intersection}}$

Soln 2 $r = \cos\theta - \sin\theta \Rightarrow \left.\begin{array}{l} x = r\cos\theta = \cos^2\theta - \frac{1}{2}\sin(2\theta) \\ y = r\sin\theta = \frac{1}{2}\sin(2\theta) - \sin^2\theta \end{array}\right\} \Rightarrow$

$2 = x + y = \cos^2\theta - \sin^2\theta = \cos(2\theta) \Rightarrow$ no solns $\Rightarrow$ $\boxed{\text{no point of intersection}}$