Your Name

Your Signature

Student ID #

Your TA's name

Your Quiz Section Label and Time

Problem	Points	Possible
1		11
2		6
3		10
4		17
5		6
Total		50

- No books allowed. You may use a scientific calculator and one $8\frac{1}{2} \times 11$ sheet of notes.
- Do not share notes.
- In order to receive credit, you must show your work and explain your reasoning (except on the "short answer" questions).
- **Place a box around $\boxed{\text{YOUR FINAL ANSWER}}$ to each question.**
- If you need more room, use the backs of the pages and indicate to the grader where to find your work.
- Raise your hand if you have a question or need more paper.

Don't open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!

1. **(11=2+3+3+3 points)** Give an example of each of the following. (*No explanation of answers needed for this problem. Be sure to explain your answers on other problems!*)

   (a) A **nonzero** vector $\mathbf{v}$ such that $\text{proj}_{\mathbf{j}}\mathbf{v} = \mathbf{0}$

   $\vec{v} = \vec{i}$ or $\vec{v} = \vec{k}$
   or any $\vec{v}$ of the form $a\vec{i} + b\vec{k}$

   (b) A vector of length 20 that is parallel to $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$. How many such vectors are there?

   There are two such vectors.
   They are $\pm \frac{20}{\sqrt{2^2+1^2+2^2}} \langle 2, -1, -2 \rangle = \pm \frac{20}{3} \langle 2, -1, -2 \rangle$
   So they are $\boxed{\langle \frac{40}{3}, -\frac{20}{3}, -\frac{40}{3} \rangle \text{ and } \langle -\frac{40}{3}, \frac{20}{3}, \frac{20}{3} \rangle}$

   (c) A vector that is perpendicular to both $\mathbf{i} - \mathbf{k}$ and $\mathbf{j} + \mathbf{k}$. How many such vectors are there?

   There are ∞-many such vectors. They all are multiples of

   $$\langle 1, 0, -1 \rangle \times \langle 0, 1, 1 \rangle = \langle +1, -1, +1 \rangle = \boxed{i - j + k}$$

   (d) Two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$ such that $|\mathbf{u} \cdot \mathbf{v}| = |\mathbf{u}||\mathbf{v}|$.

   Any two parallel vectors have this property, e.g. $\boxed{\vec{u} = \vec{v} = \vec{i}}$

2. (**6 points**) Find parametric equations for the line that contains the point $(-2, 3, 5)$ and is parallel to the planes $x + 2y + z = 4$ and $2x + 3z = 9$.

$\boxed{P_0(-2,3,5)}$ $\vec{n}_1 = \langle 1,2,1 \rangle$, $\vec{n}_2 = \langle 2,0,3 \rangle$

The direction vector of this line must be // to $\vec{n}_1 \times \vec{n}_2 = \langle 1,2,1 \rangle \times \langle 2,0,3 \rangle = \langle 6,-1,-4 \rangle$

So we can take $v = \langle 6,-1,-4 \rangle$.

The parametric eqns of the line are then

$\boxed{x = -2+6t, \quad y = 3-t, \quad z = 5-4t}$

3. **(10=3+2+2+3 points)** Consider the surface $x = y^2 + z^2 - 4y - 2z + 5$.

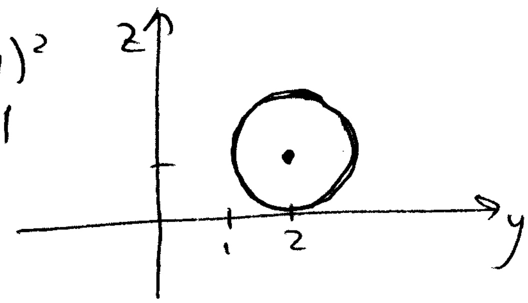
(a) Reduce this equation to one of the standard forms.

$$x = (y^2 - 4y) + (z^2 - 2z) + 5$$
$$x = (y^2 - 4y + 4) + (z^2 - 2z + 1)$$
$$\boxed{x = (y-2)^2 + (z-1)^2}$$

(b) Identify the trace of the surface in the plane $x = 1$ (i.e., Is it an ellipse, a circle, a parabola, a hyperbola, etc?) and make a sketch of it.

$x = 1 \Rightarrow 1 = (y-2)^2 + (z-1)^2$

This is a $\boxed{\text{circle}}$ of radius 1 centered at $(2,1)$

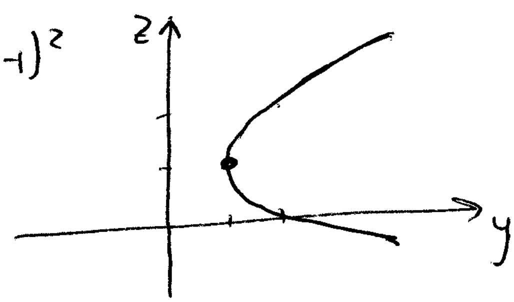


(c) Identify the trace of the surface in the plane $y = 3$. (i.e., Is it an ellipse, a circle, a parabola, a hyperbola, etc?) and make a sketch of it.

$y = 3 \Rightarrow x = (3-2)^2 + (z-1)^2$

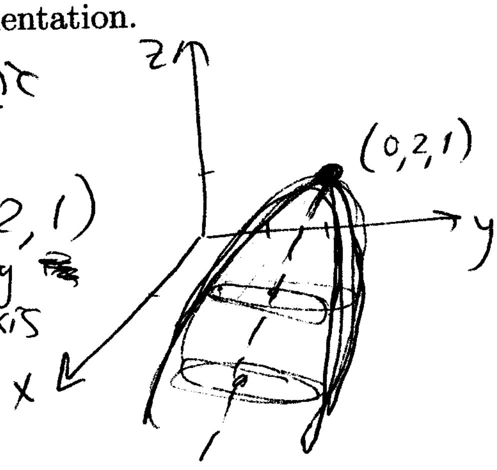
$x = 1 + (z-1)^2$

This is a parabola



(d) Identify the surface (i.e., Is this an ellipsoid, paraboloid, cone, hyperboloid of one sheet, etc?) and make a sketch of it. Your picture does not have to be drawn to scale. I am only interested in seeing the shape and orientation.

This is an elliptic paraboloid with vertex at $(0,2,1)$ and axis of symmetry parallel to the x-axis



4. **(17=4+4+5+4 points)** Consider the curve given by the vector function $\mathbf{r}(t) = \langle \cos t, \cos t, \sqrt{2}\sin t\rangle$, where $0 \le t \le 2\pi$.

(a) Compute $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$.

$\vec{r}'(t) = \langle -\sin t, -\sin t, \sqrt{2}\cos t\rangle$

$\vec{r}''(t) = \langle -\cos t, -\cos t, -\sqrt{2}\sin t\rangle$

(b) Find a parametrization of the tangent line of this curve at the point $(1/2, 1/2, \sqrt{3/2})$.

$\cos t = \frac{1}{2}, \sqrt{2}\sin t = \frac{\sqrt{3}}{\sqrt{2}} \Rightarrow \left.\begin{matrix}\cos t = \frac{1}{2} \\ \sin t = \frac{\sqrt{3}}{2}\end{matrix}\right\} \Rightarrow t = \boxed{\frac{\pi}{3}}$

$\left.\begin{matrix}\Rightarrow \vec{v} = \vec{r}'(\frac{\pi}{3}) = \langle -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2}\rangle \\ P_0\left(\frac{1}{2}, \frac{1}{2}, \sqrt{\frac{3}{2}}\right)\end{matrix}\right\} \Rightarrow$ parametric eqns of the line are

$\boxed{x = \frac{1}{2} - \frac{\sqrt{3}}{2}t, \; y = \frac{1}{2} - \frac{\sqrt{3}}{2}t, \; y = \sqrt{\frac{3}{2}} + \frac{\sqrt{2}}{2}t}$

(c) Find the curvature of this curve at the point $(1/2, 1/2, \sqrt{3/2})$.

$\vec{r}'(\frac{\pi}{3}) = \langle -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2}\rangle \Rightarrow |r'| = \sqrt{\frac{3}{4} + \frac{3}{4} + \frac{2}{4}} = \sqrt{2}$

$\vec{r}''(\frac{\pi}{3}) = \langle -\frac{1}{2}, -\frac{1}{2}, -\frac{\sqrt{2}\cdot\sqrt{3}}{2}\rangle$

$\vec{r}'(\frac{\pi}{3}) \times \vec{r}''(\frac{\pi}{3}) = \langle \frac{\sqrt{2}\cdot 3}{4} + \frac{\sqrt{2}}{4}, -\left(\frac{\sqrt{2}\cdot 3}{4} + \frac{\sqrt{2}}{4}\right), \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}\rangle$

$= \langle \sqrt{2}, -\sqrt{2}, 0\rangle \Rightarrow |r' \times r''| = \sqrt{2+2+0} = 2$

$\Rightarrow \kappa(\frac{\pi}{3}) = \frac{2}{(\sqrt{2})^3} = \boxed{\frac{1}{\sqrt{2}}}$

(d) Reparametrize this curve with respect to arc length measured from the point where t = 0 in the direction of increasing t.

$s = \int_0^t |\vec{r}'(u)|\, du = \int_0^t \sqrt{(-\sin u)^2 + (-\sin u)^2 + (\sqrt{2}\cos u)^2}\, du$

$= \int_0^t \sqrt{2\sin^2 u + 2\cos^2 u}\, du = \int_0^t \sqrt{2}\, du$

$= \sqrt{2}\cdot u \Big|_0^t = \sqrt{2}t$

$\Rightarrow t = \frac{s}{\sqrt{2}} \Rightarrow \vec{r}(t(s)) = \boxed{\langle -\sin\left(\frac{s}{\sqrt{2}}\right), -\sin\left(\frac{s}{\sqrt{2}}\right), \sqrt{2}\cos\left(\frac{s}{\sqrt{2}}\right)\rangle}$

5. (**6 points**) Find all points of intersection between the curve defined by the polar equation $r = \sec\theta + 2\tan\theta$ and the vertical line $x = 3$ or explain why there are no intersection points.

$$\left.\begin{array}{l} x = r\cos\theta \\ x = 3 \\ r = \sec\theta + 2\tan\theta \end{array}\right\} \Rightarrow 3 = (\sec\theta + 2\tan\theta)\cos\theta$$

so

$$3 = 1 + 2\sin\theta$$

$$\Rightarrow \boxed{\sin\theta = 1} \quad (*)$$

$$\Rightarrow \underline{\cos\theta = 0}.$$

But then $\sec\theta = \frac{1}{\cos\theta}$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$ are <u>undefined</u>, so these curves do <u>NOT intersect</u>

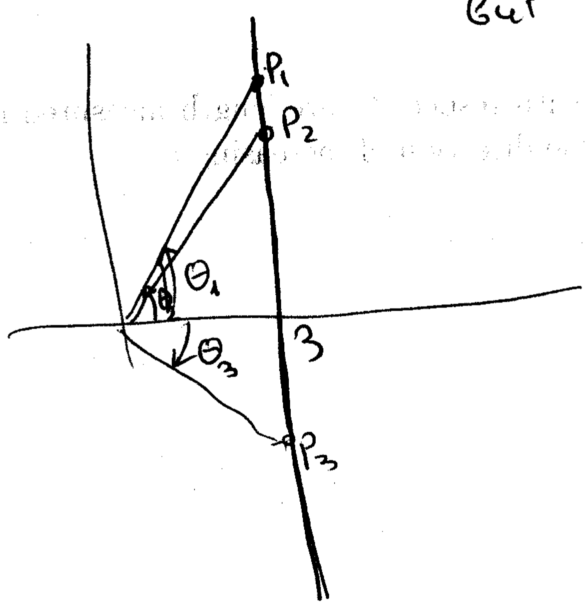
[A slightly different reasoning (starting from (*))

$\sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2} + 2k\cdot\pi$,

But for pts on the line $x = 3$,

$$\boxed{-\frac{\pi}{2} < \theta < \frac{\pi}{2}},$$

so <u>no</u> pt on the line $x=3$ satisfies $\sin\theta = 1$.



Hence the two curves do <u>NOT intersect</u>