Your Name ____________________________

Your Signature _________________________

Student ID # _____________________________

Your TA’s name ___________________________

Your Quiz Section Label and Time ___________________________

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- No books allowed. You may use a scientific calculator and one $8\frac{1}{2} \times 11$ sheet of notes.
- Do not share notes.
- In order to receive credit, you must show your work and explain your reasoning (except on the “short answer” questions).
- Place a box around **YOUR FINAL ANSWER** to each question.
- If you need more room, use the backs of the pages and indicate to the grader where to find your work.
- Raise your hand if you have a question or need more paper.

Don’t open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!
1. (11=2+3+3+3 points) Give an example of each of the following. (No explanation of answers needed for this problem. Be sure to explain your answers on other problems!)

(a) A nonzero vector \( \mathbf{v} \) such that \( \text{proj}_\mathbf{v} \mathbf{0} = 0 \)

\[
\mathbf{v} = \mathbf{i} \quad \text{or} \quad \mathbf{v} = \mathbf{k}
\]

or any \( \mathbf{v} \) of the form \( \alpha \mathbf{i} + \beta \mathbf{k} \)

(b) A vector of length 20 that is parallel to \( 2\mathbf{i} - \mathbf{j} - 2\mathbf{k} \). How many such vectors are there?

There are two such vectors.

They are \( \pm \frac{20}{\sqrt{2^2 + (-1)^2 + (-2)^2}} \mathbf{v} = \pm \frac{20}{3} \langle 2, -1, -2 \rangle \)

So they are \( \langle \frac{40}{3}, \frac{-20}{3}, \frac{-40}{3} \rangle \) and \( \langle \frac{-40}{3}, \frac{20}{3}, \frac{20}{3} \rangle \)

(c) A vector that is perpendicular to both \( \mathbf{i} - \mathbf{k} \) and \( \mathbf{j} + \mathbf{k} \). How many such vectors are there?

There are infinitely many such vectors. They all are multiples of

\[
\langle 1, 0, -1 \rangle
\]

\[
\times \langle 0, 1, 1 \rangle
\]

\[
\times \langle 1, -1, 1 \rangle = \langle i-j+k \rangle
\]

(d) Two nonzero vectors \( \mathbf{u} \) and \( \mathbf{v} \) such that \( |\mathbf{u} \cdot \mathbf{v}| = |\mathbf{u}||\mathbf{v}| \).

Any two parallel vectors have this property, e.g. \( \mathbf{u} = \mathbf{v} = \mathbf{i} \).
2. (6 points) Find parametric equations for the line that contains the point \((-2, 3, 5)\) and is parallel to the planes \(x + 2y + z = 4\) and \(2x + 3z = 9\).

\[
\text{P}(-2, 3, 5) \quad \vec{n}_1 = \langle 1, 2, 1 \rangle, \quad \vec{n}_2 = \langle 2, 0, 3 \rangle
\]

The direction vector of this line must be \(\parallel\) to \(\vec{n}_1 \times \vec{n}_2 = \langle 1, 2, 1 \rangle \times \langle 2, 0, 3 \rangle = \langle 6, -1, -4 \rangle\)

So we can take \(V = \langle 6, -1, -4 \rangle\).

The parametric equations of the line are then

\[
x = -2 + 6t, \quad y = 3 - t, \quad z = 5 - 4t
\]
3. (10=3+2+2+3 points) Consider the surface \( x = y^2 + z^2 - 4y - 2z + 5 \).

(a) Reduce this equation to one of the standard forms.

\[
\begin{align*}
X &= (y^2 - 4y) + (z^2 - 2z) + 5 \\
X &= (y^2 - 4y + 4) + (z^2 - 2z + 1) \\
\sqrt{X} &= (y - 2)^2 + (z - 1)^2
\end{align*}
\]

(b) Identify the trace of the surface in the plane \( x = 1 \) (i.e., Is it an ellipse, a circle, a parabola, a hyperbola, etc?) and make a sketch of it.

\[
\begin{align*}
x = 1 &\implies 1 = (y - 2)^2 + (z - 1)^2 \\
\text{This is a circle of radius 1 centered at (2,1)}
\end{align*}
\]

(c) Identify the trace of the surface in the plane \( y = 3 \). (i.e., Is it an ellipse, a circle, a parabola, a hyperbola, etc?) and make a sketch of it.

\[
\begin{align*}
y = 3 &\implies x = (3 - 2)^2 + (z - 1)^2 \\
x &= 1 + (z - 1)^2 \\
\text{This is a parabola.}
\end{align*}
\]

(d) Identify the surface (i.e., Is this an ellipsoid, paraboloid, cone, hyperboloid of one sheet, etc?) and make a sketch of it. Your picture does not have to be drawn to scale. I am only interested in seeing the shape and orientation.

\[
\text{This is an elliptic paraboloid} \\
\text{with vertex at (0,2,1)} \\
\text{and axis of symmetry parallel to the x-axis}
\]
4. (17=4+4+5+4 points) Consider the curve given by the vector function \( \mathbf{r}(t) = (\cos t, \cos t, \sqrt{2} \sin t) \), where \( 0 \leq t \leq 2\pi \).

(a) Compute \( \mathbf{r}'(t) \) and \( \mathbf{r}''(t) \).

\[
\mathbf{r}'(t) = \left< -\sin t, -\sin t, \sqrt{2} \cos t \right> \\
\mathbf{r}''(t) = \left< -\cos t, -\cos t, -\sqrt{2} \sin t \right>
\]

(b) Find a parametrization of the tangent line of this curve at the point \( (1/2, 1/2, \sqrt{3}/2) \).

\[
\cos t = \frac{1}{2}, \quad \sin t = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad t = \frac{\pi}{3}
\]

\[
\mathbf{p}(t) = \left< \frac{1}{2}, \frac{1}{2}, \frac{\sqrt{3}}{2} \right> + \left< -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2} \right> t = \left< \frac{1}{2} - \frac{\sqrt{3}}{2} t, \frac{1}{2} - \frac{\sqrt{3}}{2} t, \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} t \right>
\]

(c) Find the curvature of this curve at the point \( (1/2, 1/2, \sqrt{3}/2) \).

\[
\kappa = \frac{\sqrt{2} \cdot 2}{\left( \frac{1}{2} \right)^2} = \frac{2}{\frac{1}{2}^3} = \frac{8}{\sqrt{2}}
\]

(d) Reparametrize this curve with respect to arc length measured from the point where \( t = 0 \) in the direction of increasing \( t \).

\[
\ell = \int_0^t \sqrt{1 + \left( \frac{\sqrt{2} \cdot \sin u - \sqrt{2} \cos u \cdot \cos \frac{u}{2}}{\sqrt{2} \cdot \sin u + \sqrt{2} \cos u \cdot \cos \frac{u}{2}} \right)^2} \\
= \int_0^t \sqrt{2} \sin^2 u + 2 \cos^2 u \, du = \frac{t}{\sqrt{2}}
\]

\[
\Rightarrow \quad t = \frac{s}{\sqrt{2}} \quad \Rightarrow \quad \mathbf{r}(t(s)) = \left< -\sin \left( \frac{s}{\sqrt{2}} \right), -\sin \left( \frac{s}{\sqrt{2}} \right), \sqrt{2} \cos \left( \frac{s}{\sqrt{2}} \right) \right>
\]
5. (6 points) Find all points of intersection between the curve defined by the polar equation \( r = \sec \theta + 2 \tan \theta \) and the vertical line \( x = 3 \) or explain why there are no intersection points.

\[
\begin{align*}
  x &= 1 - \cos \theta \\
  x &= 3 \\
  y &= \frac{\sec \theta + 2 \tan \theta}{1 - \cos \theta} \\
\end{align*}
\]

\[
3 = (\sec \theta + 2 \tan \theta) \cos \theta
\]

\[
\begin{align*}
  3 &= 1 + 2 \sin \theta \\
  \Rightarrow \quad \sin \theta &= 1 \\
  \Rightarrow \quad \cos \theta &= 0,
\end{align*}
\]

But then \( \sec \theta = \frac{1}{\cos \theta} \) and \( \tan \theta = \frac{\sin \theta}{\cos \theta} \) are undefined, so these curves do NOT intersect.

(A slightly different reasoning (starting from \( \Theta \))

\[
\begin{align*}
  \sin \theta &= 1 \\
  \Rightarrow \quad \theta &= \frac{\pi}{2} + 2\pi \cdot n
\end{align*}
\]

But for \( P_2 \) on the line \( x = 3 \),

\[
\begin{align*}
  \frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}
\end{align*}
\]

no pt on the line \( x = 3 \) satisfies \( \theta \geq \frac{\pi}{2} \).

Hence the two curves do NOT intersect.