Math 126 Midterm #1

April 25, 2013

Name:

Section:

Instructions: This is a closed book exam. *Only scientific calculators that cannot graph, calculate derivatives, calculate integrals, make coffee, or do anything other than simple scientific calculations* are allowed. Please explain your answers to all questions clearly and succinctly. Failure to explain will result in 0 points and sadness. Please do not discuss the exam with other students until after 4 PM on Tuesday.

Time allotted: 50 minutes.

The following is for use during grading.

Problem	Points	Score
1	12	
2	12	
3	8	
4	8	

1. Let's think about spirals given by equations of the form $r = \theta^a$ in polar coordinates.

   (a) Find a spiral that passes through the point with Cartesian coordinates $(0, e)$ (i.e., find a in the equation above so that the given point lies in the plane curve determined by that relation between polar coordinates).

(b) How many such spirals (i.e., distinct values of a) are there in the preceding part? If the answer is unique, explain why. If not, find as many examples as you can. If there are infinitely many, explain why.

(c) Find the arc length of the spiral $r = \theta^2$ for the portion of the spiral between $\theta = 0$ and $\theta = 2\pi$. (Please evaluate the integral, but don't worry too much about simplifying your answer.)

2. Consider the shape determined by the equation $x^2 + y^2 = z^2$.

   (a) Draw an approximate sketch of the shape.

   (b) Show that the parametric path $((1-t)^2\cos(t), (1-t)^2\sin(t), (1-t)^2)$ lies on the shape.

(c) For which time(s) t is the speed of a particle moving along the path of part (b) minimized? (Recall that the speed is the magnitude of the velocity vector.)

3. Consider the four points $(0,0,0),(1,0,0),(2,3,0),(5,4,4)$.

   (a) Do all four points lie in a common plane? (Explain your answer unless you are happy with 0 points.)

(b) Find parametric equations for the line through $(5, 4, 4)$ that is perpendicular to the plane spanned by the three points $(0, 0, 0), (1, 0, 0), (2, 3, 0)$.

4. Projection time!

   (a) Find the projection of the vector $\mathbf{v} = \langle 0, 0, 1 \rangle$ in the direction normal to the plane

$$x - \sqrt{9}y + 10^{200}z = 0.$$

(b) Let **w** denote the projection from part (a). Suppose you have a calculator that calculates decimal expansions of numbers completely correctly up to 127 digits. Will this calculator allow you to tell the difference between **v** and **w**? Explain your answer clearly and carefully or feel the grader's wrath. (Note: you should be able to answer this even if you made silly – non-conceptual – mistakes on part (a).)