Exam I Hints and Answers

Math 126 D Spring 2011

1. (a) ANSWER: $(-2, 14, 6)$

   (b) ANSWER: $y - 2z = 2$

2. HINT: $x = r\cos\theta = (6 + \cos(6\theta))\cos\theta$, $y = r\sin\theta = (6 + \cos(6\theta))\sin\theta$, and $\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta}$.

   ANSWER: $y = -\sqrt{3}x + 10$

3. (a) HINT: $\overrightarrow{PQ} = \langle k-3, -3, 0\rangle$, $\overrightarrow{PS} = \langle -5, -4, 2\rangle$, and $|\overrightarrow{PQ} \times \overrightarrow{PS}| = \sqrt{581}$.
   ANSWER: $k = 5$

   (b) HINT: Let R be the point with coordinates (x, y, z). Then $\overrightarrow{QR} = \langle x-5, y-1, z-1\rangle$ and $\overrightarrow{QR} = \overrightarrow{PS}$.
   ANSWER: $(0, -3, 3)$

4. (a) ANSWER: $\overrightarrow{T}(t) = \langle -\frac{3}{5}\sin 3t, \frac{4}{5}, \frac{3}{5}\cos 3t\rangle$ and $\overrightarrow{N}(t) = \langle -\cos 3t, 0, -\sin 3t\rangle$

   (b) HINT: The direction vector for the line of intersection of two planes is any vector that is orthogonal to both planes.
   A vector that is orthogonal to the normal plane is $\vec{n}_1 = \overrightarrow{T}\left(\frac{\pi}{12}\right)$. A vector that is orthogonal to the osculating plane is $\vec{n}_2 = \overrightarrow{B}\left(\frac{\pi}{12}\right)$. Note that the unit normal vector is parallel to the cross-product of $\vec{n}_1 \times \vec{n}_2$. So, the unit normal vector may be used as the direction vector of the line of intersection: $\vec{v} = \overrightarrow{N}\left(\frac{\pi}{12}\right)$.
   ANSWER: $x = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}t$, $y = \frac{\pi}{3}$, $z = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}t$