

Math 126 A B and C Midterm 1 Autumn 2024

Prof. Charles Camacho

Math 126 A, B & C

Midterm 1 Exam, Autumn 2024

Print Your Full Name

Solutions

Signature

Student ID Number

Quiz Section

Instructor's Name

TA's Name

Please read these instructions!

1. Your exam contains 6 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. You are allowed a single, double-sided 8.5"x11" handwritten notesheet; the TI-30XIIS calculator; a writing utensil; and an eraser to use on the exam.
3. The exam is worth 50 points. Point values for problems vary and these are clearly indicated. You have 50 minutes for this exam.
4. Make sure to ALWAYS SHOW YOUR WORK CLEARLY. Credit is awarded to work which is clearly shown and legible. Full credit may not be awarded if work is unclear or illegible.
5. For problems that aren't sketches, place a box around your final answer to each question.
6. If you need extra space, use the last two pages of the exam. Clearly indicate on that there is more work located on the last pages, and indicate on those pages the related problem number.
7. Unless otherwise instructed, always give your answers in exact form. For example, 3π , $\sqrt{2}$, and $\ln(2)$ are in exact form; the corresponding approximations 9.424778, 1.4142, and 0.693147 are NOT in exact form.
8. Credit is awarded for correct use of techniques or methods discussed in class thus far. Partial credit may be awarded as earned. No credit is awarded for use of methods that are learned later in the course.

Problem	Total Points
1	10
2	10
3	10
4	10
5	10
Total	50

1. (10 points) Find an equation of the surface consisting of all points (x, y, z) whose distance to the point $(1, 1, -1)$ equals the distance to the plane $y = -2$. Then, identify the surface by name.

$$\sqrt{(x-1)^2 + (y-1)^2 + (z+1)^2} = \sqrt{(y+2)^2}$$

$$(x-1)^2 + (y-1)^2 + (z+1)^2 = (y+2)^2$$

$$(x-1)^2 + \cancel{y^2} - 2y + 1 + (z+1)^2 = \cancel{y^2} + 4y + 4$$

$$(x-1)^2 + (z+1)^2 = 6y + 3$$

Paraboloid

2. (10 points) Let $\mathbf{u}$ and $\mathbf{v}$ be vectors such that

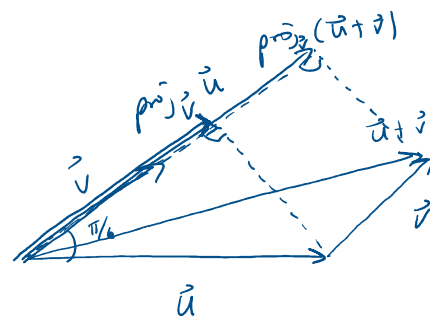
- the angle θ between $\mathbf{u}$ and $\mathbf{v}$ is acute with $\sin \theta = 1/2$,
- $\text{proj}_{\mathbf{v}} \mathbf{u} = \langle 1, 1, 2 \rangle$,
- and $\text{proj}_{\mathbf{v}} (\mathbf{u} + \mathbf{v}) = \langle 3, 3, 6 \rangle$.

Find $\text{comp}_{\mathbf{u}} \mathbf{v}$.

$$\text{comp}_{\mathbf{u}} \mathbf{v} = |\mathbf{v}| \cos \theta$$

$$= \sqrt{24} \cdot \cos \frac{\pi}{6}$$

$$= \boxed{\sqrt{24} \cdot \frac{\sqrt{3}}{2}}$$



$$\text{proj}_{\mathbf{v}} (\mathbf{u} + \mathbf{v}) = \text{proj}_{\mathbf{v}} \mathbf{u} + \mathbf{v}$$

$$\langle 3, 3, 6 \rangle = \langle 1, 1, 2 \rangle + \mathbf{v}$$

$$\langle 2, 2, 4 \rangle = \mathbf{v}$$

$$\sqrt{4+4+16} = |\mathbf{v}|$$

$$\sqrt{24} = |\mathbf{v}|$$

3. (10 points) Let $\mathbf{r}(t) = \langle \sqrt{3t^2 + 1}, \ln(2t^2), t^3 \rangle$.

Find parametric equations for the line tangent to the space curve of $\mathbf{r}(t)$ at the point $(2, \ln 2, -1)$.

$$\mathbf{r}'(t) = \left\langle \frac{1}{2\sqrt{3t^2+1}} \cdot 6t, \frac{1}{2t^2} \cdot 4t, 3t^2 \right\rangle$$

$$= \left\langle \frac{3t}{\sqrt{3t^2+1}}, \frac{2}{t}, 3t^2 \right\rangle$$

$$\mathbf{r}'(-1) = \left\langle \frac{-3}{\sqrt{4}}, \frac{2}{-1}, 3 \right\rangle = \left\langle -\frac{3}{2}, -2, 3 \right\rangle$$

$$\Rightarrow \begin{cases} x = 2 - \frac{3}{2}t \\ y = \ln 2 - 2t \\ z = -1 + 3t \end{cases}$$

4. (10 points) Find a vector function $\mathbf{r}(t)$ whose space curve is the intersection of the paraboloid

$$z = 2\left(x - \frac{1}{2}\right)^2 + 2(y-1)^2 \text{ and the plane } 2x + 4y + z = 5.$$

$$2x + 4y + 2\left(x - \frac{1}{2}\right)^2 + 2(y-1)^2 = 5$$

$$2x + 4y + 2\left(x^2 - x + \frac{1}{4}\right) + 2(y^2 - 2y + 1) = 5$$

$$\cancel{x} + \cancel{2y} + \cancel{x^2} - \cancel{x} + \frac{1}{4} + \cancel{y^2} - \cancel{2y} + 1 = \frac{5}{2}$$

$$x^2 + y^2 = \frac{5}{2} - \frac{1}{4} - 1$$

$$x^2 + y^2 = \frac{5}{4} \quad \leftarrow \text{circle radius } \frac{\sqrt{5}}{2}$$

centered at origin

$$\Rightarrow \mathbf{r}(t) = \left\langle \frac{\sqrt{5}}{2} \cos t, \frac{\sqrt{5}}{2} \sin t, 2\left(\frac{\sqrt{5}}{2} \cos t - \frac{1}{2}\right)^2 + 2\left(\frac{\sqrt{5}}{2} \sin t - 1\right)^2 \right\rangle$$

5. (10 points) Let L_1 be the line perpendicular to the plane $-2x + 3y - 5z = 8$ through the point $(1, 4, -2)$. Let L_2 be the line of intersection of the planes $x - y = 4$ and $-x - y - z = 2$. The lines are skew (you do not need to verify this).

Find equations of the two parallel planes that contain L_1 and L_2 .

$\vec{n}$ = normal vector = cross product of direction vectors of L_1 and L_2

• Direction vector of L_1 : $\vec{v}_1 = \langle -2, 3, -5 \rangle$

• Direction vector of L_2 : $\vec{v}_2$ = cross product of normal vectors to planes

Normal vector of $x - y = 4 \Rightarrow \vec{n}_1 = \langle 1, -1, 0 \rangle$

Normal vector of $-x - y - z = 2 \Rightarrow \vec{n}_2 = \langle -1, -1, -1 \rangle$

$$\vec{v}_2 = \vec{n}_1 \times \vec{n}_2 = \langle 1, 1, -2 \rangle$$

$$\vec{n} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 3 & -5 \\ 1 & 1 & -2 \end{vmatrix} = \langle -1, -9, -5 \rangle$$

• Eq. of plane containing L_1 : $\boxed{-(x-1) - 9(y-4) - 5(z+2) = 0}$

• Eq. of plane containing L_2 :

$$\begin{array}{rcl} x - y = 4 & \rightarrow & x + 3 = 4 \\ -x - y - z = 2 & & x = 1 \end{array} \quad \text{Point on plane: } (1, -3, 0)$$

$$\begin{array}{rcl} \text{Set } z = 0: & x - y = 4 \\ & -x - y = 2 \\ \hline & -2y = 6 \\ & y = -3 \end{array}$$

$$\Rightarrow \boxed{-(x-1) - 9(y+3) - 5z = 0}$$

Extra scratch paper.

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