

MATH 126–A Midterm Exam 1
Thursday October 28th

Name: _____ Quiz section: _____

7-digit UW ID: _____

Exam Instruction:

- You have 50 minutes to complete the 5–page exam. Distribute your time wisely.
- Show your work to earn full credit. If you can not figure out the final answer to a question, providing reasonable work and steps may still earn you some partial credit.
- **Do NOT write within 1 cm of the edge!** Your exam will be scanned for grading. In case you ran out of space, indicate “out of space, see scratch paper” and box your additional work in the provided scratch paper in the last page.
- Leave your answer in the exact form rather than a decimal approximation. For example, you may leave your answer as a fraction, a square root expression, an inverse trig function of an expression, etc.
- You can prepare one hand-written double-sided 8.5” × 11”–inch page of notes and bring it to the exam.
- You may use a basic calculator that can not graph, differentiate, or integrate (e.g., TI-30X IIS). **All other electronic devices (e.g., cell phone, earbuds) are forbidden.**
- **You must finish the exam independently. Giving or receiving any assistance on the exam is considered cheating, which will result in a grade of zero for the exam. There are multiple versions of the exam, do not discuss the exam questions with other students on the exam day.**
- You may tear off the last page to use it as a scratch paper. No need to turn in the scratch paper unless you have written any work to be graded.

1. (12 pts) A particle moving along a curve has position vector $\mathbf{r}(t) = \langle t, t^2 + 1, t^2 - t \rangle$.

(a) Find the time t at which the velocity vector is perpendicular to the acceleration vector.

(b) Does the curve intersect the line $x = t + 1, y = 2t + 3, z = t$? (must show work to receive credit.)

2. (12 pts) Let C be the curve of intersection of

the plane $x - 2y - z = 1$ and the cylinder $y^2 + z^2 = 4$

(a) Find a vector function $\mathbf{r}(t)$ for the curve C . Do not involve any square root function.

(b) Find a line that intersects the curve C at the point $(3, 0, 2)$ and is perpendicular to the curve at this point (i.e., the angle of intersection between the curve and the line at the point is $\pi/2$). The answer is not unique, give a parametric equation for one such line that is not parallel to the x , y , or z -axis.

3. (7 pts) Find an equation for the surface containing all the points whose distance to the point $(0, 3, 0)$ is twice its distance to the xz -plane. Simplify the equation and identify it as one of the six standard forms of quadric surfaces.

4. (7 pts) Find an equation for the plane that contains the point $P(3, 7, 3)$ and the line

$$x - 1 = \frac{y - 3}{2} = 2z.$$

5. (12 pts) Given that $\mathbf{a} = \langle 2, 1, 2 \rangle$ and $\text{comp}_{\mathbf{a}} \mathbf{b} = 5$ (i.e., the scalar projection of \mathbf{b} onto \mathbf{a} is 5).

(a) Find $\text{proj}_{\mathbf{a}} \mathbf{b}$.

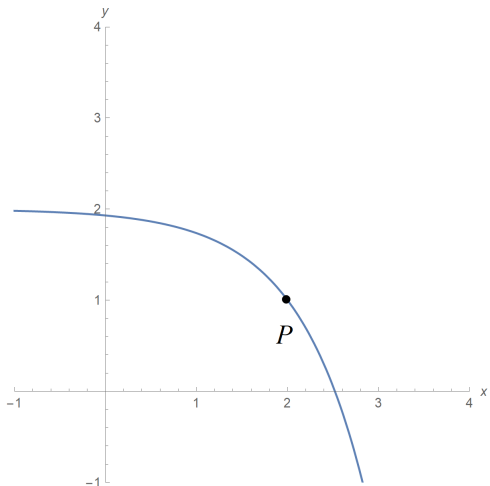
(b) Find $\mathbf{a} \cdot \mathbf{b}$.

(c) Find an example of such a vector \mathbf{b} that is not parallel to \mathbf{a} . The answer is not unique.

(d) If you are also given that $|\mathbf{a} \times \mathbf{b}| = 12$, find the angle between \mathbf{a} and \mathbf{b} .

6. (12 pts) A particle is moving along the curve in the graph below. At the point P , the velocity vector of the particle is $\mathbf{v} = \langle -3, 4 \rangle$ and it is speeding up at a rate of 15 m/sec^2 . You are also given that the curvature of the curve at the point P is $\kappa = \frac{2}{5}$.

(a) On the given graph, sketch the unit tangent vector \mathbf{T} and the unit normal vector \mathbf{N} at the point P . Then explicitly find the vectors \mathbf{T} and \mathbf{N} at the point P .



(b) Find the tangential component a_T and the normal component a_N of the acceleration vector at the point P .

(c) Use your answer in the previous parts to find the y -component of the acceleration vector at the point P .

scratch paper

(Turn in this page only if you have included any work to be graded.)