1. **[5 points per part]** For this problem, consider the following planes:

$$P_1: \quad 5x + y + 4z = 1 \qquad \text{and} \qquad P_2: \quad 10x + 2y = 3$$

(a) Find the point on P_1 closest to $(11, 3, 12)$.

Line through $(11,3,12)$ normal to P_1

$$\left.\begin{aligned} x &= 11 + 5t \\ y &= 3 + t \\ z &= 12 + 4t \end{aligned}\right\}$$

Intersection with P_1:

$5(11+5t) + 3 + t + 4(12+4t) = 1$

$55 + 25t + 3 + t + 48 + 16t = 1$

$42t = -105$

$t = \frac{-5}{2}$

$\boxed{\left(\frac{-3}{2}, \frac{1}{2}, 2\right)}$

(b) Find the acute angle of intersection between P_1 and P_2.

Angle btwn $\underbrace{\langle 5,1,4\rangle}_{\vec{a}}$ and $\underbrace{\langle 10,2,0\rangle}_{\vec{b}}$

$\vec{a}\cdot\vec{b} = 52$

$|\vec{a}| = \sqrt{42}$

$|\vec{b}| = \sqrt{104}$

$52 = \sqrt{42}\sqrt{104}\cos\theta$

$\theta = \cos^{-1}\left(\frac{52}{\sqrt{42}\sqrt{104}}\right) = \boxed{\cos^{-1}\left(\frac{13}{\sqrt{273}}\right)}$

(c) Find parametric equations for the line of intersection of P_1 and P_2.

Say $x = t$

$y = \frac{3-10t}{2} = \frac{3}{2} - 5t$

$5t + \frac{3}{2} - 5t + 4z = 1 \rightarrow 4z = \frac{-1}{2} \rightarrow z = \frac{-1}{8}$

$$\boxed{\begin{aligned} x &= t \\ y &= \frac{3}{2} - 5t \\ z &= \frac{-1}{8} \end{aligned}}$$

2. **[1 point per part]** Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in 3-space. Indicate whether each of the following expressions is a vector, a scalar, or nonsense.

You do not need to show work on this problem.

(a) $|\mathbf{u}| + \mathbf{v} \cdot \mathbf{w}$ **Vector** **Scalar** **Nonsense**

(b) $|\mathbf{u}|\mathbf{v} - |\mathbf{w}|$ **Vector** **Scalar** **Nonsense**

(c) $\mathbf{u} \cdot (\mathbf{v} \cdot \mathbf{w})$ **Vector** **Scalar** **Nonsense**

(d) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$ **Vector** **Scalar** **Nonsense**

(e) $\text{proj}_{\mathbf{u}}(\mathbf{v} \times \mathbf{w})$ **Vector** **Scalar** **Nonsense**

(f) $\text{comp}_{\mathbf{u}}(\mathbf{v} + \mathbf{w})$ **Vector** **Scalar** **Nonsense**

3. **[3 points per part]**

You do not need to show work on this problem.

(a) Give an example of two vectors $\mathbf{a}$ and $\mathbf{b}$ such that $\mathbf{a} \times \mathbf{b} = \langle 0, 6, 0 \rangle$.

$\boxed{\langle 2,0,0\rangle \times \langle 0,0,-3\rangle,}$ for example

(b) Give an example of two different vectors $\mathbf{a}$ and $\mathbf{b}$ such that $\text{proj}_{\langle 1,0,0\rangle}\mathbf{a} = \text{proj}_{\langle 1,0,0\rangle}\mathbf{b}$.

Any two vectors with the same x-component, e.g. $\boxed{\langle 1,0,0\rangle \,\&\, \langle 1,0,1\rangle}$

(c) Give an example of a vector $\mathbf{a}$ such that $\text{proj}_{\mathbf{a}}\langle 4, 5, 6\rangle = 2\mathbf{a}$.

$\boxed{\langle 2,0,0\rangle}$, for example

4. Suppose the surface $ax^2 + y^2 + 2z^2 = b$ contains the points $(2, 0, 1)$ and $(3, 5, 1)$.

(a) **[6 points]** What are a and b?

$$4a + 2 = b$$

$$9a + 25 + 2 = b$$

$$4a + 2 = 9a + 27$$

$$5a = -25$$

$$\boxed{a = -5}$$

$$-20 + 2 = b$$

$$\boxed{b = -18}$$

(b) **[2 points]** Give the name of this surface.

$$-5x^2 + y^2 + 2z^2 = -18$$

$$\frac{5x^2}{18} - \frac{y^2}{18} - \frac{z^2}{9} = 1$$

hyperboloid of two sheets

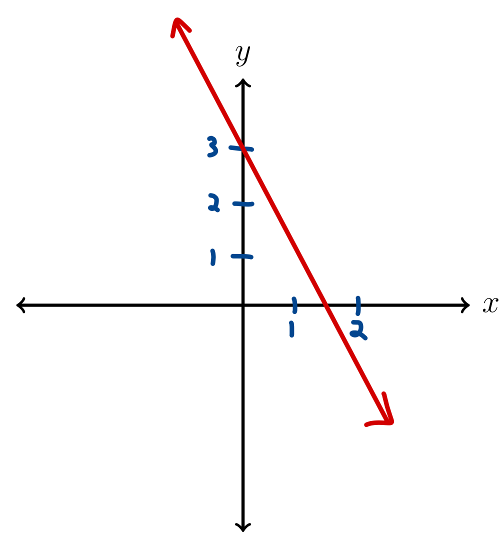
5. **[7 points]** Draw a graph of the polar curve $r = \dfrac{3}{\sin\theta + 2\cos\theta}$. Label your graph clearly.

$$r\sin\theta + 2r\cos\theta = 3$$

$$y + 2x = 3$$

$$y = -2x + 3$$

It's a line!



6. **[5 points per part]** Consider the space curve of the following vector function:

$$\mathbf{r}(t) = \langle \sin(t), \underbrace{4t+3}, \underbrace{t^2+8t} \rangle$$

(a) Find all points where the space curve intersects the plane $z = y + 9$.

$$t^2 + 8t = 4t + 3 + 9$$

$$t^2 + 4t - 12 = 0$$

$$(t+6)(t-2) = 0$$

$$t = -6 \quad \text{or} \quad t = 2$$

$$\boxed{(\sin(-6), -21, -12) \text{ and } (\sin(2), 11, 20)}$$

(b) Write parametric equations for the line tangent to the space curve at the point $(0, 3, 0)$.

$$4t+3=3$$
$$t=0$$

$$\vec{r}\,'(t) = \langle \cos(t), 4, 2t+8 \rangle$$

$$\vec{r}\,'(0) = \langle 1, 4, 8 \rangle$$

direction

$$\boxed{\begin{aligned} x &= t \\ y &= 3+4t \\ z &= 8t \end{aligned}}$$

(c) Find the curvature of the space curve at the point $(0, 3, 0)$.

$$\vec{r}\,''(t) = \langle -\sin(t), 0, 2 \rangle$$

$$\vec{r}\,''(0) = \langle 0, 0, 2 \rangle$$

$$\vec{r}\,'(0) \times \vec{r}\,''(0) = \langle 1, 4, 8 \rangle \times \langle 0, 0, 2 \rangle = \langle 8, -2, 0 \rangle$$

$$|\vec{r}\,'(0)| = 9$$

$$|\vec{r}\,'(0) \times \vec{r}\,''(0)| = \sqrt{68} = 2\sqrt{17}$$

$$K = \boxed{\frac{2\sqrt{17}}{9^3}}$$