MATH 126 C
Exam I
Autumn 2018

Name ________________________________________

Student ID #_________________ Section _________________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:___________________________________

- Your exam should consist of this cover sheet, followed by 6 problems on 5 pages. Check that you have a complete exam.
- Pace yourself. You have 50 minutes to complete the exam and there are 5 pages. Try not to spend more than 10 minutes on each page.
- Unless otherwise indicated, **show all your work and justify your answers**.
- Unless otherwise indicated, your answers should be exact values rather than decimal approximations. (For example, $\frac{\pi}{4}$ is an exact answer and is preferable to its decimal approximation 0.7854.)
- You may use a **TI 30XII S** calculator and one 8.5×11-inch sheet of handwritten notes. **All other calculators, electronic devices, and sources are forbidden**.
- **Do not write within one centimeter of the edge of the page.** Your exam will be scanned for grading.
- If you need more room, ask your TA for extra paper, put your name on it, and **tell the grader where to look for your solution**.
- The use of headphones or earbuds during the exam is not permitted.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.
- You are not allowed to use your phone for any reason during this exam. **Turn your phone off and put it away for the duration of the exam**.

GOOD LUCK!

1. (10 points) Consider two planes:

$$x - y + 2z = 3 \text{ and } -2x + 3y - z = 5.$$

(a) Compute the angle between the planes.

(b) Find the point $(x, y, z)$ at which the **line of intersection of the two planes** intersects the plane $x = 2$.

(c) Give parametric equations for the line of intersection of the two planes.

2. (4 points) Give the equation of **any** plane that is orthogonal to the line

$$\mathbf{r}(t) = \langle 1 + 2t, -3t, 4 - t \rangle$$

and **does not contain** the origin. (There are infinitely many correct answers. You need to give only one correct answer.)

3. (6 points) A point $P$ has polar coordinates $\left(7, \frac{5\pi}{6}\right)$.

(a) Give the Cartesian coordinates $(x, y)$ of the point $P$.

(b) Give a different polar coordinate representation $(r, \theta)$ of the point $P$ in which $r < 0$ and $\theta < 0$.

(c) Give a different polar coordinate representation $(r, \theta)$ of the point $P$ in which $r < 0$ and $\theta > 0$.

4. (10 points)

(a) For each of the following planes, find and identify the trace (if it exists) of the quadric surface $y = x^2 - 2z^2$ in that plane. (You do not need to show any work.)

Choose your answers from the following list:

| | | | |
|---|---|---|---|
| circle | ellipse | hyperbola | parabola |
| point | line | pair of lines | does not exist |

i. the plane $x = k$

ii. the plane $y = k$, $k \neq 0$

iii. the plane $y = 0$

iv. the plane $z = k$

(b) Identify the surface $y = x^2 - 2z^2$. (You do not need to show any work.)

Choose your answer from the following list:

| | | |
|---|---|---|
| elliptic cylinder | parabolic cylinder | hyperbolic cylinder |
| paraboloid | ellipsoid | hyperbolic paraboloid |
| cone | hyperboloid of one sheet | hyperboloid of two sheets |

(c) Let $\ell$ be the line through the points $(0, 3, 0)$ and $(5, -1, \sqrt{13})$. Find all points $(x, y, z)$ of intersection of the line $\ell$ with the surface $y = x^2 - 2z^2$. (Show all work.)

5. (10 points) The acceleration vector of a spaceship is

$$\mathbf{a}(t) = \langle 2t, 0, -\sin(t) \rangle \quad \text{for all } t \geq 0$$

and the specified initial velocity and position are

$$\mathbf{v}(0) = \langle 0, 0, 1 \rangle \quad \text{and} \quad \mathbf{r}(0) = \langle 1, 2, 300 \rangle.$$

(a) Find the velocity function of the spaceship.

(b) Find the tangential component of the acceleration at time $t$.

(c) Compute the ship's position at $t = \dfrac{\pi}{2}$.

6. (10 points) Suppose a 3-D curve is represented by the vector function

$$\mathbf{r}(t) = \langle t^2 - 1, \frac{1}{3}t^3 - 2t, t^2 - 2 \rangle, \ -\infty < t < \infty.$$

(a) There are two points $(x, y, z)$ at which the curve intersects the $yz$-plane. Find them.

(b) Let $\ell_1$ be the line tangent to the curve at one of the points you found in (a) and $\ell_2$ be the line tangent to the curve at the other point you found in (a). Find the point $(x, y, z)$ at which $\ell_1$ and $\ell_2$ intersect or show that they do not.