Exam I Answers

Math 126 D, E, & F & F Autumn 2017

Version 1: In #1, A is the point $(0, -1, 4)$.

1. (a) $\cos^{-1}\left(\dfrac{23}{\sqrt{29}\sqrt{41}}\right)$

   (b) $\sqrt{165}$

   (c) $\dfrac{35}{\sqrt{165}}$

2. (a) C

   (b) $\dfrac{-2}{\pi+1}$

3. (a) Since $\dfrac{dx}{dt} = 1 + 3t^2$ is always positive, the x-coordinate is always increasing. Thus, the object moves from left to right.

   (b) $t = \dfrac{1}{2}$

   (c) $\dfrac{\pi^2}{8}$

4. (a) There are many correct answers.
   Here's one:
   $\mathbf{a} = \langle 10, -3, 4\rangle$ and $\mathbf{i} = \langle 1, 0, 0\rangle$.
   The vector $\mathbf{a} \times \mathbf{i} = \langle 0, 4, 3\rangle$ is orthogonal to $\mathbf{a}$ and $|\mathbf{a} \times \mathbf{i}| = 5$.
   Let $\mathbf{v} = \dfrac{101}{5}(\mathbf{a} \times \mathbf{i}) = \dfrac{101}{5}\langle 0, 4, 3\rangle$.
   Then $\mathbf{v}$ is orthogonal to $\mathbf{a}$ and has length 101.

   (b) There are many correct answers.
   Here's one:
   Choose y and z so that $2y^2 = \sin^2 t$ and $z^2 = \cos^2 t$ so that $2y^2 + z^2 = 1$ for all t.
   That is, let $y = \dfrac{\sin t}{\sqrt{2}}$ and $z = \cos t$.
   Then let $x = y^2 + z^2 = \dfrac{\sin^2 t}{2} + \cos^2 t$.
   The curve of intersection is then

$$x = \frac{\sin^2 t}{2} + \cos^2 t, y = \frac{\sin t}{\sqrt{2}}, z = \cos t.$$

Version 2: In #1, A is the point $(1, 0, -3)$.

1. (a) $\cos^{-1}\left(\dfrac{16}{\sqrt{41}\sqrt{26}}\right)$

   (b) $\dfrac{9\sqrt{10}}{2}$

(c) $\dfrac{7}{\sqrt{10}}$

2. (a) D

   (b) $\dfrac{2\pi+1}{2}$

3. (a) Since $\dfrac{dx}{dt} = 1 + 3t^2$ is always positive, the x-coordinate is always increasing. Thus, the object moves from left to right.

   (b) $t = \dfrac{1}{2}$

   (c) $\dfrac{\pi^2}{8}$

4. (a) There are many correct answers.
   Here's one:
   $\mathbf{a} = \langle 2, -10, 3 \rangle$ and $\mathbf{i} = \langle 1, 0, 0 \rangle$.
   The vector $\mathbf{a} \times \mathbf{i} = \langle 0, 3, 10 \rangle$ is orthogonal to $\mathbf{a}$ and $|\mathbf{a} \times \mathbf{i}| = \sqrt{109}$.
   Let $\mathbf{v} = \dfrac{105}{\sqrt{109}}(\mathbf{a} \times \mathbf{i}) = \dfrac{105}{\sqrt{109}}\langle 0, 3, 10 \rangle$.
   Then $\mathbf{v}$ is orthogonal to $\mathbf{a}$ and has length 105.

   (b) There are many correct answers.
   Here's one:
   Choose y and z so that $y^2 = \sin^2 t$ and $5z^2 = \cos^2 t$ so that $y^2 + 5z^2 = 1$ for all t.
   That is, let $y = \sin t$ and $z = \dfrac{\cos t}{\sqrt{5}}$.
   Then let $x = y^2 + z^2 = \sin^2 t + \dfrac{\cos^2 t}{5}$.
   The curve of intersection is then

$$x = \sin^2 t + \frac{\cos^2 t}{5}, y = \sin t, z = \frac{\cos t}{\sqrt{5}}.$$