Math 126 G - Autumn 2017
Midterm Exam Number One
October 24, 2017

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Signature: wwwwwwwwwwwwwwwwwwwn   Section: ???

1	12	12
2	10	10
3	10	10
4	13	13
5	15	15
Total	60	60

- This exam consists of FIVE problems on FIVE pages, including this cover sheet.
- Show all work for full credit. Show no work for zero credit.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- Write all of your work on the exam itself. If you use the back of the page, please indicate that you have done so!
- You may use a TI-30X IIS on this exam. No other electronic devices are allowed.
- Draw a box around your final answer to each problem.
- You may use one hand-written double-sided 8.5″ by 11″ page of notes.
- You have 50 minutes to complete the exam.

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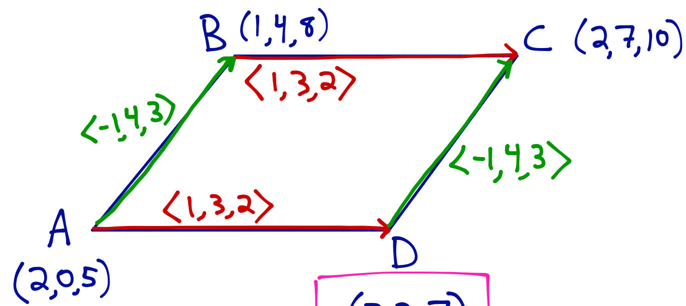
1. **[4 points per part]** $ABCD$ is a parallelogram, with diagonals AC and BD.

Here are some coordinates:

$$A = (2,0,5) \qquad B = (1,4,8) \qquad C = (2,7,10)$$

(a) What are the coordinates of D?



$\boxed{(3,3,7)}$

(b) Find the area of the parallelogram $ABCD$.

$$\text{area} = \left|\langle 1,3,2\rangle \times \langle -1,4,3\rangle\right| = \left|\langle 1,-5,7\rangle\right|$$

$$= \sqrt{1^2+5^2+7^2} = \sqrt{75} = \boxed{5\sqrt{3}}$$

(c) Find the equation of the plane containing this parallogram.

Using cross product as normal vector: $\boxed{x - 5y + 7z = 37}$

using any of A, B, C, or D

2. **[10 points]** Write an equation for the ellipsoid centered at $(2, 4, -1)$ and containing the points $(-6, 1, -1)$, $(2, -1, -1)$, and $(4, 3, 2)$.

$$\frac{(x-2)^2}{a^2} + \frac{(y-4)^2}{b^2} + \frac{(z+1)^2}{c^2} = 1$$

$$0 + \frac{25}{b^2} + 0 = 1 \rightarrow b = 5$$

$$\frac{64}{a^2} + \frac{9}{25} + 0 = 1 \rightarrow a = 10$$

$$\frac{4}{100} + \frac{1}{25} + \frac{9}{c^2} = 1 \rightarrow c^2 = \frac{225}{23}$$

$$\boxed{\frac{(x-2)^2}{100} + \frac{(y-4)^2}{25} + \frac{23(z+1)^2}{225} = 1}$$

3. **[10 points]** I have some secret vectors **u** and **v**.

- $\text{proj}_{\mathbf{v}}\mathbf{u} = \langle 3, -1, 2 \rangle$
- $\text{proj}_{\mathbf{u}}\mathbf{v} = \langle 5, 1, -1 \rangle$

So, what's **u**?

So $\vec{u}$ points in the direction $\langle 5, 1, -1 \rangle$

$\vec{u} = \langle 5t, t, -t \rangle$ for some t.

$$\text{proj}_{\vec{v}}\vec{u} = \text{proj}_{\langle 3,-1,2\rangle}\vec{u} = \frac{\langle 5t, t, -t\rangle \cdot \langle 3,-1,2\rangle}{|\langle 3,-1,2\rangle|^2}\langle 3,-1,2\rangle = \langle 3,-1,2\rangle$$

same direction

$$\frac{\langle 5t, t, -t\rangle \cdot \langle 3,-1,2\rangle}{|\langle 3,-1,2\rangle|^2} = 1$$

$$15t - t - 2t = 14$$

$$t = \frac{7}{6}$$

$$\boxed{\vec{u} = \left\langle \frac{35}{6}, \frac{7}{6}, \frac{-7}{6} \right\rangle}$$

4. Consider the polar curve $r = 1 - 6\cos(\theta)$.

(a) **[4 points]** Find all intersections of the curve with the x-axis.

$\theta = 0 \rightarrow r = 1-6 = -5 \rightarrow \boxed{(-5,0)}$

$\theta = \pi \rightarrow r = 1+6 = 7 \rightarrow \boxed{(-7,0)}$

$r = 0 \rightarrow \theta = \cos^{-1}\left(\frac{1}{6}\right) \rightarrow \boxed{(0,0)}$

(b) **[9 points]** Find the x-coordinates of all points on the curve at which the tangent line is horizontal.

$$\frac{dr}{d\theta} = 6\sin\theta$$

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta} = 0 \rightarrow 6\sin^2\theta + (1-6\cos\theta)\cos\theta = 0$$

$$6 - 6\cos^2\theta + \cos\theta - 6\cos^2\theta = 0$$

$$-12\cos^2\theta + \cos\theta + 6 = 0$$

or use quadratic formula...

$$(-4\cos\theta + 3)(3\cos\theta + 2) = 0$$

$\cos\theta = \frac{3}{4}$ or $\cos\theta = \frac{-2}{3}$

$r = 1 - 6\left(\frac{3}{4}\right)$ $\qquad$ $r = 1 - 6\left(\frac{-2}{3}\right)$

$r = \frac{-7}{2}$ $\qquad$ $r = 5$

$x = r\cos\theta$ $\qquad$ $x = r\cos\theta$

$\boxed{x = \frac{-21}{8}}$ $\qquad$ $\boxed{x = \frac{-10}{3}}$

5. **[5 points per part]**

(a) Write a vector function $\mathbf{r}(t)$ whose space curve is the intersection of the surfaces

$$x + y - z = 1 \qquad \text{and} \qquad x = z^2.$$

Set $z = t$

$x = t^2$

$y = 1 - x + z = -t^2 + t + 1$

$$\boxed{\vec{r}(t) = \langle t^2, -t^2 + t + 1, t \rangle}$$

(b) Let P_1 and P_2 be the intersections of this space curve with the plane $y = -11$. Find parametric equations for the lines tangent to the curve at P_1 and P_2.

$-t^2 + t + 1 = -11$

$t^2 - t - 12 = 0$

$(t-4)(t+3) = 0$

$t = 4$ or $t = -3$

$\vec{r}\,'(t) = \langle 2t, -2t+1, 1 \rangle$

point: $(9, -11, -3)$

direction: $\langle -6, 7, 1 \rangle$

$$\boxed{\begin{aligned} x &= 9 - 6t \\ y &= -11 + 7t \\ z &= -3 + t \end{aligned}}$$

point: $(16, -11, 4)$

direction: $\langle 8, -7, 1 \rangle$

$$\boxed{\begin{aligned} x &= 16 + 8t \\ y &= -11 - 7t \\ z &= 4 + t \end{aligned}}$$

(c) Are the lines you found in part (b) parallel, intersecting, or skew?

Clearly not parallel: $\frac{-6}{8} \neq \frac{7}{-7}$ → Long answer: $9 - 6s = 16 + 8t$

$-11 + 7s = -11 - 7t \rightarrow s = -t$

$-3 + s = 4 + t \rightarrow s = \frac{7}{2},\ t = \frac{-7}{2}$

check: $9 - 6\left(\frac{7}{2}\right) \stackrel{?}{=} 16 + 8\left(\frac{-7}{2}\right)$

Yes! $\boxed{\text{Intersecting}}$

Short answer: they both lie in the plane $x + y - z = 1$, so they can't be skew.

So $\boxed{\text{intersecting.}}$