MATH 126, SECTIONS C AND D, AUTUMN 2016, MIDTERM I
OCTOBER 20, 2016

Name Solutions

TA/Section

Instructions.

- There are 4 questions. The exam is out of 40 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. **Hand in your notes with your exam paper.**
- You may use a TI 30X IIS calculator. Even if you have a calculator, give me exact answers. ($\frac{2\ln 3}{\pi}$ is exact, 0.7 is an approximation for the same number.)
- **Show your work.** If I cannot read or follow your work, I cannot grade it. You may not get full credit for a right answer if your answer is not justified by your work. If you continue at the back of a page, make a note for me. Please $\boxed{\text{BOX}}$ your final answer.

Question	points
1	
2	
3	
4	
Total	

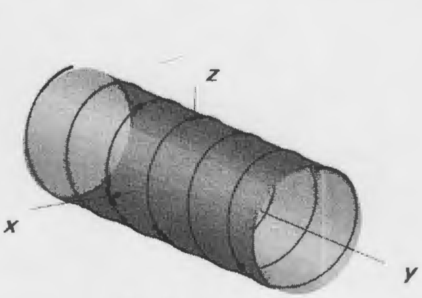
1. (a) (6 points) Match the following curves and surfaces with their graphs. Give your answer in the form G7, letter of the surface followed by the number of the curve. To get the point, you have to have both the surface and the curve correct under the corresponding picture.

Surfaces:

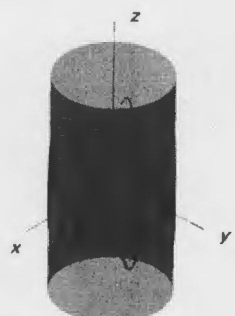
A. $16x^2 + z^2 = 16$	**B.** $x^2 + z^2 = 1$	**C.** $x^2 + y^2 = z^2$
D. $x^2 + y^2 = 1$	**E.** $z = \sin(2y)$	**F.** $x^2 + y^2 + z^2 = 1$

Curves:

1. $\mathbf{r}(t) = \langle \sin(6t), t, \cos(6t) \rangle$	**2.** $\mathbf{r}(t) = \langle \cos(t)\cos(16t), \cos(t)\sin(16t), \sin(t) \rangle$
3. $\mathbf{r}(t) = \left\langle \sqrt{t}, 3t, \sin(6t) \right\rangle$	**4.** $\mathbf{r}(t) = \langle \sin(2t), 3 - 2(\cos(2t) + \sin(2t)), 4\cos(2t) \rangle$
5. $\mathbf{r}(t) = \langle t\sin(16t), t\cos(16t), t \rangle$	**6.** $\mathbf{r}(t) = \langle \sin(2t), \cos(2t), \cos(12t) \rangle$



B1



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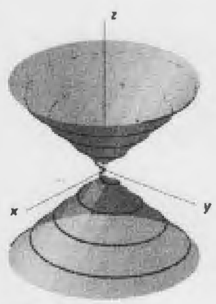
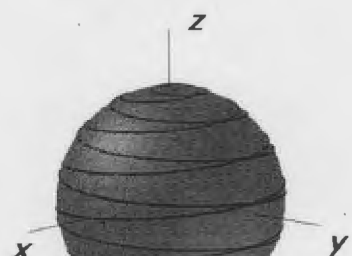
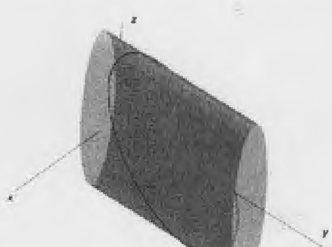


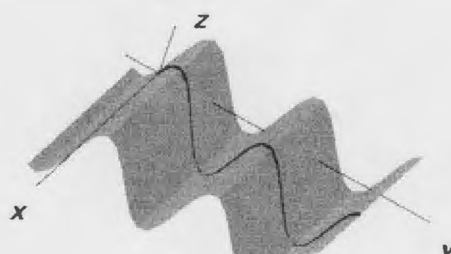
C5



F2



A4



E3

(b) (4 points) Find the equation of the tangent line to the curve given by

$$\mathbf{r}(t) = (t^3 + 2t + 5)\mathbf{i} - e^t\mathbf{j} + \sin(3t)\mathbf{k}$$

at the point $(5, -1, 0)$.

$-1 = -e^t$ when $\boxed{t=0}$

$\vec{r}\,'(t) = (3t^2+2)\vec{i} - e^t\vec{j} + 3\cos(3t)\vec{k}$

$\vec{r}\,'(0) = 2\vec{i} - \vec{j} + 3\vec{k} = \langle 2,-1,3 \rangle$

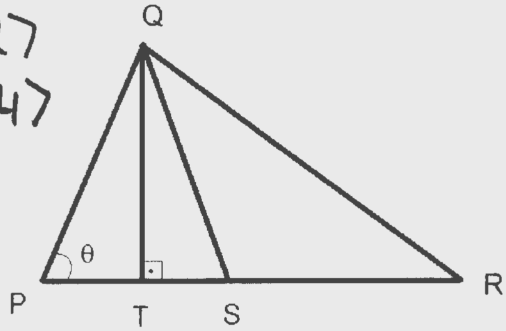
Line Equation

$\vec{r}_\ell(t) = \langle 5,-1,0 \rangle + \langle 2,-1,3 \rangle t$

$\boxed{\vec{r}_\ell(t) = \langle 5+2t, -1-t, 3t \rangle}$

2. A triangle has its vertices at $P(1,2,3)$, $Q(0,4,1)$ and $R(5,1,-1)$.

(a) What is the angle θ?



$$\vec{PQ} = \langle 0-1, 4-2, 1-3 \rangle = \langle -1, 2, -2 \rangle$$

$$\vec{PR} = \langle 5-1, 1-2, -1-3 \rangle = \langle 4, -1, -4 \rangle$$

$$\cos\theta = \frac{\vec{PQ}\cdot\vec{PR}}{\|\vec{PQ}\|\,\|\vec{PR}\|} = \frac{-4-2+8}{\sqrt{1+4+4}\,\sqrt{16+1+16}}$$

$$\theta = \cos^{-1}\left(\frac{2}{3\sqrt{33}}\right)$$

(b) Find the coordinates of the point S, midway between points P and R.

$$\frac{1}{2}\vec{PR} = \langle 2, -\tfrac{1}{2}, -2 \rangle$$

$$S = (1+2,\ 2-\tfrac{1}{2},\ 3-2) = (3, \tfrac{3}{2}, 1)$$

(c) The angle at point T is a right angle as shown. Find the coordinates of the point T.

$$\vec{PT} = \text{proj}_{\vec{PR}}\vec{PQ} = \frac{\langle -1,2,-2\rangle\cdot\langle 4,-1,-4\rangle}{\langle 4,-1,-4\rangle\cdot\langle 4,-1,-4\rangle}\langle 4,-1,-4\rangle$$

$$= \frac{2}{33}\langle 4,-1,-4\rangle = \langle \tfrac{8}{33}, \tfrac{-2}{33}, \tfrac{-8}{33} \rangle$$

$$T = \left(1+\tfrac{8}{33},\ 2-\tfrac{2}{33},\ 3-\tfrac{8}{33}\right) = \left(\tfrac{41}{33}, \tfrac{64}{33}, \tfrac{91}{33}\right)$$

(d) What is the area of the triangle?

$$\frac{1}{2}\|\vec{PQ}\times\vec{PR}\| \quad \text{or} \quad \frac{1}{2}\|\vec{PR}\|\,\|\vec{QT}\|$$

$$\vec{PQ}\times\vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & -2 \\ 4 & -1 & -4 \end{vmatrix} = (-8-2)\vec{i} - (4+8)\vec{j} + (1-8)\vec{k}$$

$$= \langle -10, -12, -7 \rangle$$

$$\text{Area} = \frac{1}{2}\sqrt{100+144+49} = \frac{1}{2}\sqrt{293}$$

3. (10 points) Decide if the following lines intersecting or skew.

$$\mathbf{r}_1(t) = (2t+1)\mathbf{i} + (5-7t)\mathbf{j} + (3t+2)\mathbf{k}$$

$$\mathbf{r}_2(t) = (2t-3)\mathbf{i} + (3t-11)\mathbf{j} + (23-6t)\mathbf{k}$$

If the are skew, find the distance between them. If they are intersecting, find the equation of the plane which contains them.

$\vec{r}_1(t) = \vec{r}_2(s)$?

$2t+1 = 2s-3$ $\quad\quad$ $5-7t = 3s-11$ $\quad\quad$ $3t+2 = 23-6s$

$2t = 2s-4$

$t = s-2 \rightsquigarrow 5-7(s-2) = 3s-11$

$5-7s+14 = 3s-11$

$30 = 10s$

$\boxed{\begin{matrix} 3 = s \\ 1 = t \end{matrix}} \rightsquigarrow 3(1)+2 \stackrel{?}{=} 23-6(3)$

$5 = 5$

The lines intersect at $\vec{r}_1(1) = \vec{r}_2(3) = \langle 3, -2, 5 \rangle$.

Point on the plane $(3, -2, 5)$

Normal for the plane

$$\vec{n} = \begin{vmatrix} i & j & k \\ 2 & -7 & 3 \\ 2 & 3 & -6 \end{vmatrix} = (42-9)\vec{i} - (-\cancel{18}12-6)\vec{j} + (6+14)\vec{k}$$

$$= \langle 33, 18, 20 \rangle$$

Plane Equation

$$33(x-3) + 18(y+2) + 20(z-5) = 0$$

OR $\quad 33x + 18y + 20z = 99 - 36 + 100 = 163$

4. (a) (6 points) Decide if the following are TRUE or FALSE. Explain your answer briefly.

i. For any two vectors **u** and **v**, $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$.

FALSE $\quad \vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$

ii. For any two vectors **u** and **v**, $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = 0$.

TRUE $\quad \vec{u} \times \vec{v}$ is perpendicular to both $\vec{u}$ and $\vec{v}$

iii. The curve with the vector equation $\mathbf{r}(t) = t^3\mathbf{i} + (2t^3 - 2)\mathbf{j} + 7\mathbf{k}$ is a line.

TRUE $\quad$ It is the line of intersection of the two planes $y = 2x - 2$ and $z = 7$

(b) (4 points) Identify the surface given by the equation

$$9x^2 - 36x + 4y^2 - 36z^2 + 72z = 36$$

Make a sketch including the name of the surface and basic information.

completing the squares:

$$9(x^2 - 4x + 4) + 4y^2 - 36(z^2 - 2z + 1) = 36 + 36 - 36$$

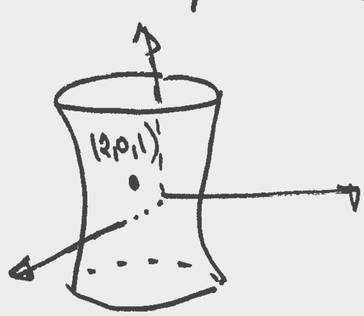
$$9(x-2)^2 + 4y^2 - 36(z-1)^2 = 36$$

$$\frac{(x-2)^2}{4} + \frac{y^2}{9} - (z-1)^2 = 1$$

hyperboloid of one sheet

axis $x=2, y=0$ parallel to z-axis

center at $(2,0,1)$



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Surfaces:

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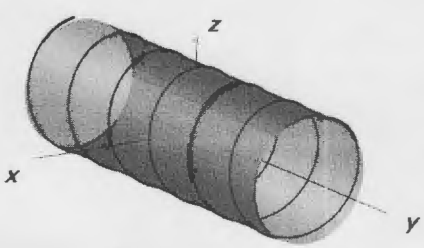
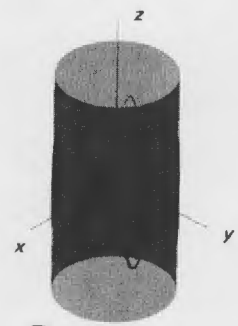


F1



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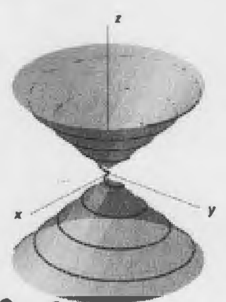
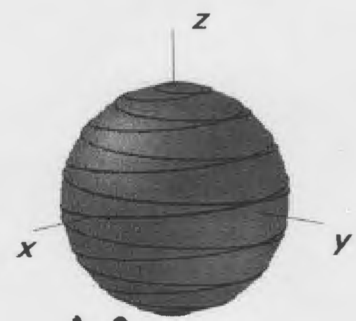
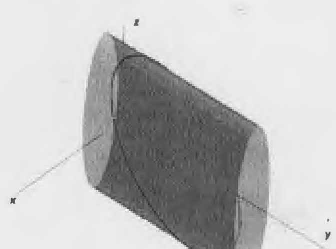


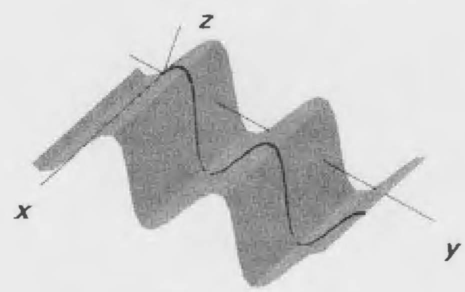
B3



A2



C6



E5

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TRUE $\vec{u} \times \vec{v}$ is perpendicular to both $\vec{u}$ & $\vec{v}$

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TRUE It's the line of intersection of the planes $z = 7$ and $y = 2x - 2$

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SAME