

1. [8 points] Answer the following questions. You need not show work or explain your answers.

- (a) [4 points] In this problem, \mathbf{u} , \mathbf{v} , and \mathbf{w} are vectors, and a , b and c are scalars. For each expression below, decide if it is a vector (**V**), a scalar (**S**), or nonsense (**N**).

	Circle one:
$(\mathbf{u} \cdot \mathbf{v})\mathbf{w}$	<input checked="" type="radio"/> V <input type="radio"/> S <input type="radio"/> N
$\frac{ \mathbf{v} }{\mathbf{v}}$	<input type="radio"/> V <input type="radio"/> S <input checked="" type="radio"/> N
$\text{comp}_{\mathbf{w}}(\mathbf{v} + b\mathbf{u})$	<input type="radio"/> V <input checked="" type="radio"/> S <input type="radio"/> N
$a\mathbf{u} \times (b\mathbf{v} \cdot c\mathbf{w})$	<input type="radio"/> V <input type="radio"/> S <input checked="" type="radio"/> N

- (b) [3 points] In this problem, \mathbf{u} , \mathbf{v} , and \mathbf{w} are **non-zero** vectors in 3-space, and no two of them are parallel or perpendicular to each other. For each statement below, decide if it is always true (**T**), always false (**F**), or only sometimes true (**S**).

	Circle one:
$\text{comp}_{\mathbf{w}}(\mathbf{v} + \mathbf{u})$ is a positive scalar	<input type="radio"/> T <input type="radio"/> F <input checked="" type="radio"/> S
$\text{proj}_{\mathbf{w}}(\mathbf{v} + \mathbf{u})$ is parallel to \mathbf{w}	<input checked="" type="radio"/> T <input type="radio"/> F <input type="radio"/> S
$\mathbf{u} \cdot (\mathbf{w} \times (-\mathbf{w}))$ is zero	<input checked="" type="radio"/> T <input type="radio"/> F <input type="radio"/> S

- (c) [1 points] Give an example of a **nonzero** vector \mathbf{v} such that $\text{proj}_{\mathbf{k}}\mathbf{v} = \mathbf{0}$

$$\vec{v} = \langle 1, 1, 0 \rangle$$

2. [12 points] Let α denote the plane $2x + y - 2z = 2$. Let A, B, C denote the points where the plane α intersects the x -axis, the y -axis, and the z -axis, respectively.

(a) [3 points] Find the coordinates of the points A, B , and C .

$$\begin{aligned} A & (1, 0, 0) \\ B & (0, 2, 0) \\ C & (0, 0, -1) \end{aligned}$$

(b) [3 points] Find a vector equation for the line through A which is parallel to the line BC .

$$\begin{aligned} \vec{r}(t) &= \vec{OA} + t \vec{BC} \\ &= \langle 1, 0, 0 \rangle + t \langle 0, -2, -1 \rangle \end{aligned}$$

$$\vec{r}(t) = \langle 1, -2t, -t \rangle$$

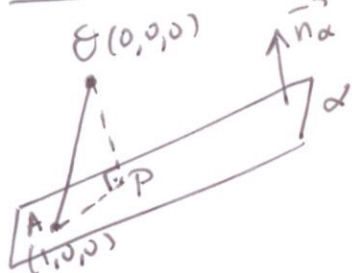
$$\text{OR } \vec{r}(t) = \langle 1, 2t, t \rangle$$

if we use \vec{CB} instead of \vec{BC}

(c) [6 points] Find the distance from the origin $O(0,0,0)$ to the plane α specified above. Show work.

There are multiple solutions. For example:

Sol. 1:



Project the vector \vec{OA} onto \vec{n}_α

$$\vec{OA} = \langle 1, 0, 0 \rangle, \quad \vec{n}_\alpha = \langle 2, 1, -2 \rangle$$

$$\vec{OA} \cdot \vec{n}_\alpha = 2 \Rightarrow \text{comp}_{\vec{n}_\alpha} \vec{OA} = \frac{2}{\|\vec{n}_\alpha\|} = \frac{2}{\sqrt{4+1+4}}$$

$$\text{distance} = |\text{comp}_{\vec{n}_\alpha} \vec{OA}| = \boxed{\frac{2}{3}}$$

Sol. 2: The line $OP \perp \alpha$ has direction $\vec{n}_\alpha = \langle 2, 1, -2 \rangle$

so it has parametric eq's: $x = 0 + 2s, y = 0 + s, z = 0 - 2s$
 $x = 2s, y = s, z = -2s$

Intersecting this line with α : $2x + y - 2z = 2$ we set:

$$2(2s) + s - 2(-2s) = 2 \Rightarrow s = 2/9$$

Point P has coord. $x = 2s = 4/9, y = s = 2/9, z = -2s = -4/9$

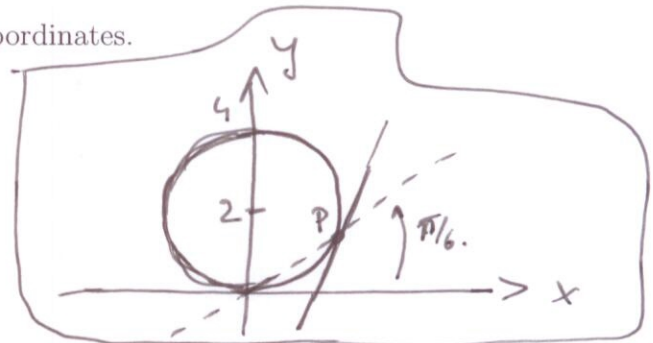
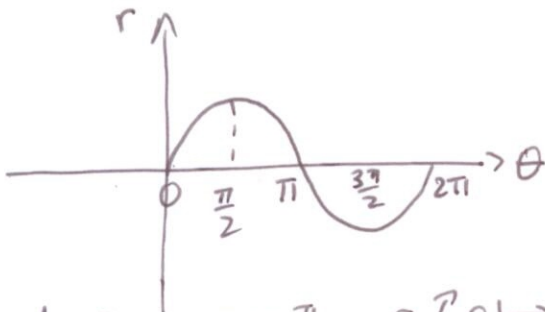
$$\Rightarrow P(4/9, 2/9, -4/9)$$

$$\text{dist.} = \|\vec{OP}\| = \sqrt{\left(\frac{4}{9}\right)^2 + \left(\frac{2}{9}\right)^2 + \left(-\frac{4}{9}\right)^2} = \frac{\sqrt{16+4+16}}{\sqrt{81}} = \frac{6}{9} = \boxed{\frac{2}{3}}$$

3. [10 points] Let C denote the polar curve

$$r = 4 \sin(\theta).$$

- (a) [4 points] Sketch the graph of C in xy -coordinates.



As θ goes $0 \rightarrow \frac{\pi}{2}$, $r \nearrow 0 \rightarrow 4$
 $\theta: \frac{\pi}{2} \rightarrow \pi$, $r \searrow 4 \rightarrow 0$. } \Rightarrow circle of radius 2 centered at $(0, 2)$.

(As θ goes $\pi \rightarrow 2\pi$, we get the same points again)

- (b) [6 points] Find the equation $y = mx + b$ of the tangent line to the curve C at $\theta = \frac{\pi}{6}$.
 Show work.

$$x = r \cos(\theta) = (4 \sin \theta) \cos \theta = 2 \sin(2\theta)$$

$$y = r \sin(\theta) = (4 \sin \theta) \sin \theta = 4 \sin^2(\theta)$$

$$\frac{dx}{d\theta} = 2 \cos(2\theta) \cdot 2 = 4 \cos(2\theta), \text{ so } \frac{dx}{d\theta} \Big|_{\theta=\pi/6} = 4 \cos\left(\frac{\pi}{3}\right) = 4 \cdot \frac{1}{2} = 2$$

$$\frac{dy}{d\theta} = 4 (2 \sin \theta \cdot \cos \theta) = 4 \sin(2\theta), \text{ so } \frac{dy}{d\theta} \Big|_{\theta=\pi/6} = 4 \sin\left(\frac{\pi}{3}\right) = 4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}$$

$$\text{slope} = \frac{dy}{dx} \Big|_{\theta=\pi/6} = \frac{\frac{dy}{d\theta} \Big|_{\theta=\pi/6}}{\frac{dx}{d\theta} \Big|_{\theta=\pi/6}} = \frac{2\sqrt{3}}{2} \Rightarrow \boxed{m = \sqrt{3}}$$

$$\text{Point: } \begin{aligned} x &= 2 \sin(2\theta) \Rightarrow x = 2 \sin\left(\frac{\pi}{3}\right) = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3} \\ y &= 4 \sin^2(\theta) \Rightarrow y = 4 \sin^2\left(\frac{\pi}{6}\right) = 4 \left(\frac{1}{2}\right)^2 = 1 \end{aligned} \Rightarrow \boxed{P(\sqrt{3}, 1)}$$

$$\text{Tan line at } P: y = \sqrt{3}(x - \sqrt{3}) + 1 = \sqrt{3}x - 3 + 1$$

$$\boxed{y = \sqrt{3}x - 2}$$

