1. **[8 points]** Answer the following questions. You need not show work or explain your answers.

(a) **[4 points]** In this problem, $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors, and a, b and c are scalars. For each expression below, decide if it is a vector (**V**), a scalar (**S**), or nonsense (**N**).

	Circle one:		
$(\mathbf{u}\cdot\mathbf{v})\mathbf{w}$	V	S	N
$\dfrac{\lvert\mathbf{v}\rvert}{\mathbf{v}}$	V	S	N
$\text{comp}_{\mathbf{w}}(\mathbf{v}+b\mathbf{u})$	V	S	N
$a\mathbf{u}\times(b\mathbf{v}\cdot c\mathbf{w})$	V	S	N

(b) **[3 points]** In this problem, $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are **non-zero** vectors in 3-space, and no two of them are parallel or perpendicular to each other. For each statement below, decide if it is always true (**T**), always false (**F**), or only sometimes true (**S**).

	Circle one:		
$\text{comp}_{\mathbf{w}}(\mathbf{v}+\mathbf{u})$ is a positive scalar	T	F	S
$\text{proj}_{\mathbf{w}}(\mathbf{v}+\mathbf{u})$ is parallel to $\mathbf{w}$	T	F	S
$\mathbf{u}\cdot(\mathbf{w}\times(-\mathbf{w}))$ is zero	T	F	S

(c) **[1 points]** Give an example of a **nonzero** vector $\mathbf{v}$ such that $\text{proj}_{\mathbf{k}}\mathbf{v}=\mathbf{0}$

$\vec{v} = \langle 1, 1, 0 \rangle$

2. [**12 points**] Let α denote the plane $2x + y - 2z = 2$. Let A, B, C denote the points where the plane α intersects the x-axis, the y-axis, and the z-axis, respectively.

(a) [**3 points**] Find the coordinates of the points A, B, and C.

$A\ (1,0,0)$
$B\ (0,2,0)$
$C\ (0,0,-1)$

(b) [**3 points**] Find a vector equation for the line through A which is parallel to the line BC.

$$\vec{r}(t) = \vec{OA} + t\,\vec{BC}$$
$$= \langle 1,0,0\rangle + t\,\langle 0,-2,-1\rangle$$

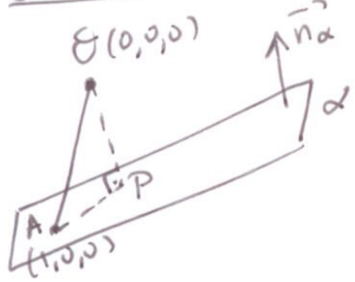
$\boxed{\vec{r}(t) = \langle 1, -2t, -t\rangle}$ OR $\boxed{\vec{r}(t) = \langle 1, 2t, t\rangle}$

If we use $\vec{CB}$ instead of $\vec{BC}$

(c) [**6 points**] Find the distance from the origin $O(0,0,0)$ to the plane α specified above. Show work.

There are multiple solutions. For example:

SOL. 1:



Project the vector $\vec{OA}$ onto $\vec{n}_\alpha$

$\vec{OA} = \langle 1,0,0\rangle$, $\vec{n}_\alpha = \langle 2,1,-2\rangle$

$\vec{OA}\cdot\vec{n}_\alpha = 2 \Rightarrow \text{comp}_{\vec{n}_\alpha}\vec{OA} = \dfrac{2}{\|\vec{n}_\alpha\|} = \dfrac{2}{\sqrt{4+1+4}}$

distance $= |\text{comp}_{\vec{n}_\alpha}\vec{OA}| = \boxed{\dfrac{2}{3}}$

SOL. 2: The line $OP \perp \alpha$ has direction $\vec{n}_\alpha = \langle 2,1,-2\rangle$

so it has parametric eq's: $x = 0+2s$, $y = 0+s$, $z = 0-2s$
$x = 2s$, $y = s$, $z = -2s$.

Intersecting this line with α: $2x+y-2z = 2$ we get:

$2(2s) + s - 2(-2s) = 2 \Rightarrow s = 2/9$

Point P has coord. $x = 2s = 4/9$, $y = s = \frac{2}{9}$, $z = -2s = -\frac{4}{9}$

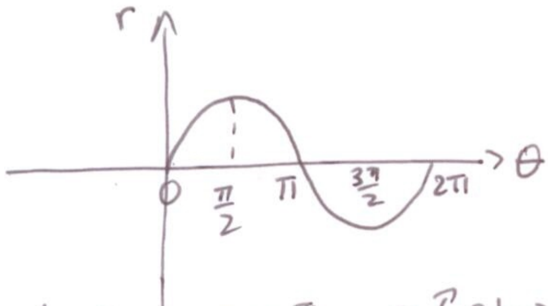
$\Rightarrow P(4/9, 2/9, -4/9)$

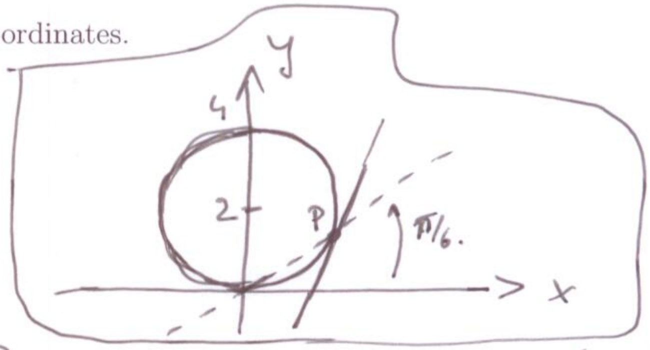
dist. $= \|\vec{OP}\| = \sqrt{\left(\frac{4}{9}\right)^2 + \left(\frac{2}{9}\right)^2 + \left(\frac{-4}{9}\right)^2} = \dfrac{\sqrt{16+4+16}}{\sqrt{81}} = \dfrac{6}{9} = \boxed{\dfrac{2}{3}}$

3. [**10 points**] Let C denote the polar curve

$$r = 4\sin(\theta).$$

(a) [**4 points**] Sketch the graph of C in xy-coordinates.





As θ goes $0 \mapsto \frac{\pi}{2}$, $r \nearrow 0 \mapsto 4$
$\theta: \frac{\pi}{2} \mapsto \pi$, $r \searrow 4 \mapsto 0$. $\Big\}$ $\Rightarrow$ circle of radius 2 centered at $(0,2)$.

(As θ goes $\pi \mapsto 2\pi$, we get the same points again)

(b) [**6 points**] Find the equation $y = mx + b$ of the tangent line to the curve C at $\theta = \dfrac{\pi}{6}$.
Show work.

$$x = r\cos(\theta) = (4\sin\theta)\cos\theta = 2\sin(2\theta)$$
$$y = r\sin(\theta) = (4\sin\theta)\sin\theta = 4\sin^2(\theta)$$

$$\frac{dx}{d\theta} = 2\cos(2\theta)\cdot 2 = 4\cos(2\theta), \text{ so } \left.\frac{dx}{d\theta}\right|_{\theta=\pi/6} = 4\cos\left(\frac{\pi}{3}\right) = 4\frac{1}{2} = 2$$
$$\frac{dy}{d\theta} = 4\,(2\sin\theta\cdot\cos\theta) = 4\sin(2\theta), \text{ so } \left.\frac{dy}{dx}\right|_{\theta=\pi/6} = 4\sin\left(\frac{\pi}{3}\right) = 4\frac{\sqrt{3}}{2} = 2\sqrt{3}$$

$$\text{slope} = \left.\frac{dy}{dx}\right|_{\theta=\pi/6} = \frac{\left.\frac{dy}{d\theta}\right|_{\theta=\pi/6}}{\left.\frac{dx}{d\theta}\right|_{\theta=\pi/6}} = \frac{2\sqrt{3}}{2} \Rightarrow \boxed{m = \sqrt{3}}$$

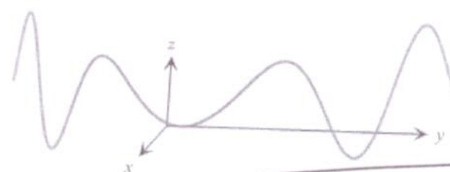
Point: $x = 2\sin(2\theta) \Rightarrow x = 2\sin(\pi/3) = 2\frac{\sqrt{3}}{2} = \sqrt{3}$
$y = 4\sin^2(\theta) \Rightarrow y = 4\sin^2(\pi/6) = 4\left(\frac{1}{2}\right)^2 = 1$ $\Big) \Rightarrow \boxed{P(\sqrt{3}, 1)}$

Tan line at P: $y = \sqrt{3}(x - \sqrt{3}) + 1 = \sqrt{3}x - 3 + 1$

$$\boxed{y = \sqrt{3}x - 2}$$

4. [**12 points**] A portion of the path followed by a rollercoaster can be parameterized by the vector function:

$$\mathbf{r}(t) = \langle -20t^2, 30t + 6, 10\sin(t^2) \rangle$$



(a) [**3 points**] Compute $\mathbf{r}'(t)$ and $\mathbf{r}'(0)$

$\vec{r}'(t) = \langle -40t, 30, 10\cos(t^2)\cdot 2t \rangle = \boxed{\langle -40t, 30, 20t\cos(t^2) \rangle}$ $\vec{r}'(t)$

$\boxed{\vec{r}'(0) = \langle 0, 30, 0 \rangle}$

(b) [**3 points**] Find parametric equations for the tangent line to this path at $t = 0$.

$\vec{r}(0) + s\,\vec{r}'(0) = \langle 0, 6, 0 \rangle + s\langle 0, 30, 0 \rangle$

$$\boxed{\begin{aligned} x &= 0 \\ y &= 6 + 30s \\ z &= 0 \end{aligned}}$$

(c) [**6 points**] Find the curvature of the rollercoaster's path at $t = 0$. Show work.

$\vec{r}''(t) = \langle -40, 0, \underbrace{20\cos(t^2)} + \underbrace{20t(-\sin(t^2))\cdot 2t}_{0.} \rangle$

$\vec{r}''(0) = \langle -40, 0, 20 \rangle$

$\vec{r}'(0) \times \vec{r}''(0) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 30 & 0 \\ -40 & 0 & 20 \end{vmatrix} = \vec{i}\cdot 600 - \vec{j}\cdot 0 + \vec{k}\cdot 1200 = \langle 600, 0, 1200 \rangle$

$\|\vec{r}'(0) \times \vec{r}''(0)\| = \sqrt{600^2 + 1200^2} = \sqrt{600^2 + (2\times 600)^2} = 600\sqrt{1+4} = 600\sqrt{5}$

$\kappa(0) = \dfrac{\|\vec{r}'(0) \times \vec{r}''(0)\|}{\|\vec{r}'(0)\|^3} = \dfrac{600\sqrt{5}}{(30)^3} = \dfrac{6\cancel{00}\sqrt{5}}{270\cancel{00}} = \dfrac{2\sqrt{5}}{90} = \dfrac{\sqrt{5}}{45}$

$$\boxed{\kappa(0) = \frac{\sqrt{5}}{45}}$$

5. [**8 points**] Circle the correct answers (no explanation needed).

Consider the surface $z^2 = 3x^2 + 2y^2$.

(a) Describe the traces (cross-sections) of this surface parallel to the given plane.

(i) Traces parallel to the yz-plane (when x is fixed) are: $z^2 - 2y^2 = \text{constant}$

PARABOLAS HYPERBOLAS ELLIPSES

(ii) Traces parallel to the xz-plane (when y is fixed) are: $z^2 - 3x^2 = \text{constant}$.

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(iii) traces parallel to the xy-plane (when z is fixed) are: $3x^2 + 2y^2 = \text{constant}$

PARABOLAS HYPERBOLAS ELLIPSES

(b) Circle the name of the surface given by $z^2 = 3x^2 + 2y^2$:

CONE, SPHERE, ELLIPSOID
Parabolic CYLINDER, Hyperbolic CYLINDER, Elliptical CYLINDER,
HYPERBOLOID, Elliptic PARABOLOID, Hyperbolic PARABOLOID,
NONE of the above

$$\frac{z^2}{6} = \frac{x^2}{2} + \frac{y^2}{3}$$

Form: $$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$