MATH 126 B/C MIDTERM 1 Autumn 2011

Oct 25, 2011

NAME: SIGNATURE:

STUDENT ID #: TA SECTION:

Problem	Number of points	Points obtained
1	10	
2	10	
3	10	
4	10	
Total	40	

Instructions:

- Your exam consists of FOUR problems. Please check that you have all four of them.
- No books or notebooks allowed; you may use an A4 double-sided, handwritten sheet of notes *for personal use* (do not share).
- Place a box around your final answer to each question.
- No *graphing* calculators allowed (scientific calculators OK).
- **Answers with little or no justification may receive no credit.**
- **Answers obtained by guess-and-check work will receive little or no credit, even if correct.**
- Read problems *carefully.*
- Raise your hand if you have a question.
- If you need more space, use additional blank sheets which will be provided by your TA. It is your responsibility to have him/her staple the additional sheets to your exam before you turn it in.
- Please turn off cell phones. GOOD LUCK!

Problem 1. (10 pts) Find the distance of the point $P(2,1,4)$ to the plane that passes through the points $Q(1,0,0), R(0,2,0)$, and $S(0,0,3)$.

Solution. The plane contains the vectors $u := QR =< -1,2,0 >$ and $v := QS =< -1,0,3 >$. Thus the distance is the absolute value of the scalar projection of $w := QP =< 1,1,4 >$ in the direction $u \times v$. A quick calculation reveals

$$u \times v = 6\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}.$$

Thus, the required distance is given by

$$\left|\text{Comp}_{u\times v} w\right| = \frac{17}{\sqrt{36+9+4}} = \frac{17}{7}.$$

Problem 2. (10 pts) Find the equation of the plane that is perpendicular to the plane $x - y + z = 2$ and contains the line with symmetric equations $x = 2y = 3z$.

Solution. We need to find two vectors on the plane. One is given by the normal vector of the given plane $P(1, -1, 1)$. We get the other two from the line $Q(0, 0, 0)$ and $R(6, 3, 2)$. Let $u := PQ =< -1, 1, -1 >$ and $v := PR =< 5, 4, 1 >$. Thus we can take the normal vector $n = u \times v =< 5, -4, -9 >$. Since the plane passes through $(0, 0, 0)$, the equation is

$$5x - 4y - 9z = 0.$$

Problem 3. (10 pts) Consider the parametric curve $x = \exp(\cos(t))$, $y = \exp(\sin(t))$, where $0 \le t < 2\pi$ is the parameter.

(i) For what values of t are the tangents horizontal ? For what values are the tangents vertical?

Solution. We have

$$\frac{dy}{dt} = \cos(t)\exp(\sin(t)), \quad \frac{dx}{dt} = -\sin(t)\exp(\cos(t)), \quad \frac{dy}{dx} = -\frac{\cos(t)}{\sin(t)}\exp(\sin(t) - \cos(t)).$$

Tangent horizontal at $t = \{\pi/2, 3\pi/2\}$ and vertical at $t = \{0, \pi\}$.

(ii) Write down the equation of the tangent line to this curve at $t = \pi/4$.

Solution. Slope is -1. Thus the line is

$$y = -x + 2\exp(1/\sqrt{2}).$$

Problem 4. (10 pts) Consider the polar curve

$$r + \frac{c}{r} = \sin(\theta) + \cos(\theta)$$

where c is some constant.

(i) Show that for $c > 1/2$ this equation has no solution.

Solution. We write it as $r^2 + c = r\cos\theta + r\sin\theta$. That gives us

$$x^2 + y^2 + c = x + y.$$

Completing the square we get

$$(x - 1/2)^2 + (y - 1/2)^2 = 1/2 - c.$$

No solution when $c > 1/2$ since the left side is positive (or zero) while the right side is negative.

(ii) Show that for $c \leq 1/2$ the above curve is a circle. What is the center and the radius of this circle?

Solution. Center at $(1/2, 1/2)$ and radius $\sqrt{\frac{1}{2} - c}$.

Extra sheet.