These problems use the techniques of section 5 except for differentiation and integration of series. Each problem can be derived from the basic series given in Examples 4.2.

(a) In problems 1-6, find the Taylor series for \( f(x) \) based at \( b \). Your answer should have one Sigma (\( \Sigma \)) sign. On some problems you might want to describe the coefficients using a multi-part notation as in Example 5.5.

(b) Then write the solution in expanded form: \( a_0 + a_1(x-b) + a_2(x-b)^2 + \ldots \) where you write at least the first three non-zero terms explicitly.

(c) Then give an interval \( I \) where the Taylor series converges.

Note that there are some hints below.

1. \( f(x) = \cos(3x^2) \) based at \( b = 0 \).
2. \( f(x) = \sin^2(x) \) based at \( b = 0 \).
3. \( f(x) = e^{4x-5} \) based at \( b = 2 \).
4. \( f(x) = \sin(x) \) based at \( b = \frac{\pi}{6} \).
5. \( f(x) = \frac{1}{4x-5} - \frac{1}{3x-2} \) based at \( b = 0 \).
6. \( f(x) = \frac{x}{(2x+1)(3x-1)} \) based at \( b = 1 \).

7. The “\( \sinh \)” and “\( \cosh \)” functions are used, for example, in electrical engineering, and are defined by \( \sinh(x) = (e^x - e^{-x})/2 \), and \( \cosh(x) = (e^x + e^{-x})/2 \). Do questions (a) and (b) above for the function \( h(x) = 2 \sinh(3x) - 4 \cosh(3x) \) based at \( b = 0 \).

8. Find the 6\(^{th} \) degree Taylor polynomial for \( f(x) = \sin(3x-5) \) based at \( b = 0 \), without differentiating.

Hints:
Change the base from \( b \) to \( 0 \) by substituting \( u = x - b \).
Be sure that the terms in your answers are numbers (coefficients) times powers of \( x - b \).
Use the double angle formula in problem 2.
Use partial fractions in problem 6.
Use the addition formulae for \( \sin(A \pm B) \) in problems 4 and 8.