Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name (check one)

Jonah Ostroff Zhixu Su James Zhang

TA's Name

- CHECK that your exam contains 8 problems on 6 double-sided pages, including this cover sheet. There is one blank page at the front and two blank pages at the back reserved for scratch work or extra space.
- This exam is closed book. You may use one $8\frac{1}{2}$" $\times$ 11" sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Write your answers in the provided blanks.**
- If you need more room, use the back of the first page or either side of the last page **and indicate that you have done so.**
- **Do not write within 1 centimeter of the edge of the page.**
- Raise your hand if you have a question.

Problem	Total Points	Score
1	10	
2	12	
3	12	
4	13	
5	12	

Problem	Total Points	Score
6	13	
7	14	
8	14	
Total	100	

You may use this page for scratch-work.

All work on this page will be ignored unless you write & circle "see first page" below a problem.

1. *(2 points each)* Each of the following multiple choice problems has one correct answer. Circle it. You do not need to show any reasoning.

   (a) If $|\mathbf{b}| = -2\,\mathrm{comp}_{\mathbf{a}}\mathbf{b}$, then what's the angle between $\mathbf{a}$ and $\mathbf{b}$?

   (i) $30°$ (ii) $60°$ (iii) $120°$ (iv) $150°$

   (b) Suppose $\overrightarrow{AB} \times \overrightarrow{AC} = \langle 2, -1, 2\rangle$. What's the area of $\triangle ABC$?

   (i) 1 (ii) 1.5 (iii) 2 (iv) 3

   (c) Which of the following points is on the line $\mathbf{r}(t) = \langle 2 - t, t, -1 + 2t\rangle$?

   (i) $(1, 1, 1)$. (ii) $(1, 1, 0)$. (iii) $(-1, 1, 2)$. (iv) $(2, 1, 1)$.

   (d) What's the intersection between the planes $2x - y + 3z = 3$ and $4x - 2y + 6z = 5$?

   (i) a point (ii) a line (iii) a plane (iv) nothing

   (e) What is the trace (cross section) of the surface $x^2 - 2x - y^2 + z^2 = 0$ in the plane $z = 1$?

   (i) a parabola (ii) a circle (iii) a hyperbola (iv) two lines

2. *(12 points)* Consider the space curve

$$\mathbf{r}(t) = \langle t, 1 + \sin t, 2t - \cos t \rangle.$$

Find the equation of the plane that passes through the point $P(1, 3, -4)$ and contains the tangent line to the curve $\mathbf{r}(t)$ at $t = 0$.

Plane: ____________________________________

3. *(12 points)* Let $f(x, y) = \ln(x+1) + \tan^{-1}(y) - \cos(xy)$. Find all of the first and second partial derivatives of f.

$f_x(x, y) =$ ____________________

$f_y(x, y) =$ ____________________

$f_{xx}(x, y) =$ ____________________

$f_{xy}(x, y) =$ ____________________

$f_{yy}(x, y) =$ ____________________

4. *(13 points)* Find the absolute minimum and maximum values of the function

$$f(x, y) = x^3 + 2xy - y^2 - x$$

over the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 2)$.

Absolute minimum: ____________________

Absolute maximum: ____________________

5. *(12 points)* Reverse the order of integration and evaluate the integral:

$$\int_{e}^{e^2} \int_{\ln y}^{2} (e^x - ex)^5 \, dx \, dy$$

Answer: ______________________

6. *(13 points)* Find the volume of the solid that lies below the cone $z = 2 + \sqrt{x^2 + y^2}$, above the xy-plane, and is enclosed by the cylinder $x^2 + (y-1)^2 = 1$.

   You may find the following the trig identities helpful:

$$\cos^2(t) = \frac{1 + \cos(2t)}{2} \qquad \text{and} \qquad \sin^2(t) = \frac{1 - \cos(2t)}{2}$$

Volume: ______________________

7. For this problem, let $f(x) = \ln(2x-1)$.

   (a) *(5 points)* Find the third Taylor polynomial, $T_3(x)$, for the function f based at $b = 1$.

   $T_3(x) =$ ______________________________

   (b) *(4 points)* Use $T_3(x)$ to approximate the value of $\ln(1.04)$.

   $\ln(1.04) \approx$ ______________________________

   (c) *(5 points)* Use Taylor's inequality to find an upper bound (as sharp as possible) for the error in your approximation in part (b).

   Upper bound: ______________________________

8. For this problem, you may use the following basic Taylor series:

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \quad e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad \sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \quad \cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$$

For parts (a)–(c), let $f(x) = x \arctan(2x^2)$

(a) *(6 points)* Give the Taylor series for f based at $b = 0$ using one sigma sign.

Taylor series: ______________________________

(b) *(3 points)* Find the largest open interval on which this Taylor series converges.

Interval: ____________________

(c) *(5 points)* Find $T_8(x)$, the 8^{th} Taylor polynomial for f based at $b = 0$, and use it to approximate $\int_0^1 f(x)\,dx$.

$T_8(x) =$ ______________________________

$\int_0^1 f(x)\,dx \approx$ ____________________

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