Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- CHECK that your exam contains 8 problems on 6 double-sided pages, including this cover sheet. There is one blank page at the front and two blank pages at the back reserved for scratch work or extra space.
- This exam is closed book. You may use one $8\frac{1}{2}$" × 11" sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the back of the first page or either side of the last page **and indicate that you have done so**. If you *still* need more room, ask for more scratch paper.
- **Do not write within 1 centimeter of the edge of the page.**
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	

Problem	Total Points	Score
6	12	
7	14	
8	14	
Total	100	

You may use this page for scratch-work.

All work on this page will be ignored unless you write & circle "see first page" below a problem.

1. *(6 points per part)*

   (a) Find parametric equations for the line of intersection of the planes $2x - y + 3z + 4 = 0$ and $-x + y - z = 0$.

   Parametric equations: ____________________

   (b) Find the intersection point of the line you found in part (a) with the plane $x+2y+z = 12$.

   Intersection point: $(x, y, z) =$ ____________________

2. *(4 points per part)* Consider the curve given by the position function $\mathbf{r}_1(t) = \langle \ln(t), t^2 + 5, 3t \rangle$ for $t > 0$.

   (a) Find the curvature at $t = 1$.

   $\kappa(1) =$ ________________

   (b) Find the (x, y, z) point on the curve at which the tangent line is orthogonal to the plane $x + 8y + 6z = 7$.

   $(x, y, z) =$ ________________

   (c) Find the **acute** angle of intersection of $\mathbf{r}_1(t)$ with the curve $\mathbf{r}_2(u) = \langle 0, u^3 - u, \sqrt{u^2 + 5} \rangle$.

   (Give your answer rounded to the nearest degree.)

   Angle $\approx$ ________________ degrees

3. *(6 points per part)* Consider the function $f(x, y) = x^2 + xy^2 - y$.

   (a) Find all the saddle point(s) of f in $\mathbb{R}^2$. Justify your answer.

   Saddle point(s): ______________________________

   (b) Let D be the closed region bounded by $x = y^2$ and $x = 1$. Find the absolute maximum and absolute minimum values of the function f on the region D, and the points where these extrema occur.

   Absolute (global) maximum on D: $f($____, ____$) =$ _____

   Absolute (global) minimum on D: $f($____, ____$) =$ _____

4. (*12 points*) Consider the surface $x^2 - (y+1)^2 - 3z^2 = 2$.

   Find the point(s) on the surface that are closest to the point $A(0, 3, 1)$.

   For full credit, you must show work to justify your answer.

Closest point(s): ______________________

5. *(6 points per part)* The parts of this question are not related.

   (a) The region D is bounded above by the curve $y = \dfrac{1}{x}$, below by the line $y = \dfrac{1}{2}(x-1)$ and to left by the line $x = 1$.

   Set up the integral $\displaystyle\iint_D f(x,y)\,dA$ using **two different orders of integration**.

   You may have to split the region.

One way: ____________________

Another way: ____________________

   (b) Evaluate $\displaystyle\int_0^1 \int_{y^4}^{\sqrt{y}} \left(\sqrt{x} - y^2\right)\,dx\,dy$.

Answer: ____________________

6. *(12 points)* Find the volume of the region that is inside the ellipsoid $x^2 + y^2 + 4z^2 = 25$ and above the cone $z = \sqrt{x^2 + y^2}$.

Volume = ______________________________

7. Let $f(x) = \sqrt{(2x+1)^3}$.

   (a) *(6 points)* Find the second Taylor polynomial $T_2(x)$ for f based at $b = 4$.

   $T_2(x) =$ ________________________________

   (b) *(3 points)* Use your answer to part (a) to approximate $\sqrt{9.1^3}$.

   $\sqrt{9.1^3} \approx$ ________________________________

   (c) *(5 points)* Use Taylor's inequality to find an upper bound (as sharp as possible) for the error in your approximation in part (b).

   Upper bound: ________________________________

8. For this problem, let $f(x) = x\sin(x^4) + \dfrac{x^2}{4 - x^3}$.

   (a) *(6 points)* Find $T_{10}(x)$, the 10th Taylor polynomial for f based at $b = 0$.

   $T_{10}(x) =$ ______________________________

   (b) *(3 points)* Find the largest open interval on which the Taylor series for f based at $b = 0$ converges.

   Interval: ______________________________

   (c) *(5 points)* Find $f^{2024}(0)$. (That is, the 2024th derivative of f at 0.) Give an exact answer.

   $f^{(2024)}(0) =$ ______________________________

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