Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- CHECK that your exam contains 8 problems on 6 double-sided pages, including this cover sheet. There is one blank page at the front and two blank pages at the back reserved for scratch work or extra space.
- This exam is closed book. You may use one $8\frac{1}{2}$" × 11" sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around $\boxed{\textbf{YOUR FINAL ANSWER}}$ to each question.**
- If you need more room, use the back of the first page or either side of the last page **and indicate that you have done so**. If you *still* need more room, ask for more scratch paper.
- **Do not write within 1 centimeter of the edge of the page.**
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	

Problem	Total Points	Score
6	12	
7	14	
8	14	
Total	100	

You may use this page for scratch-work.

All work on this page will be ignored unless you write & circle "see first page" below a problem.

1. (*4 points per part*) For parts (a)–(c), consider the plane $\mathcal{P} : 2x + y + 3z = 2$ and the line $L :\ x = 1 - 4t,\ y = 2t,\ z = 5 + t$.

   (a) Find the acute angle between the line L and the line $y = 2, z = 6$.

   (b) Find the equation of the plane that contains the line L and is perpendicular to the plane $\mathcal{P}$.

   (c) Find two points A and B on the plane $\mathcal{P}$ such that the vector $\overrightarrow{AB}$ is perpendicular to the line L.

2. (*4 points per part*) For parts (a)–(c), a particle is moving along a space curve with initial position vector $\mathbf{r}(0) = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$. The velocity vector of the particle is

$$\mathbf{v}(t) = 3\cos(t)\,\mathbf{i} + 4t\,\mathbf{j} + \sin(t)\,\mathbf{k}$$

(a) Find the position vector function $\mathbf{r}(t)$.

(b) Find the equation of a surface that contains the space curve $\mathbf{r}(t)$.

(c) Find the curvature of the space curve $\mathbf{r}(t)$ at $t = \pi$.

3. (*12 points*) Let f be the function given by:

$$f(x, y) = x^2 - y^2 - 2xy + 4y,$$

and let A, B, and C be the points $(0, 0)$, $(0, 4)$ and $(4, 0)$. Find the global minimum and maximum of f in the filled triangle ABC.

4. (*12 points*) Find a point on the surface of equation $z = x^2 - y^2$ whose tangent plane is perpendicular to the line of equation

$$\begin{aligned} x &= t \\ y &= 3t \\ z &= -2t. \end{aligned}$$

5. (*12 points*) Find the volume of the solid below the surface

$$z = \frac{2x + y + 1}{\sqrt{x^2 + y^2}}$$

and above the region in the xy-plane

$$D = \{(x, y) \mid x \geq 0, y \geq 0, 1 \leq x^2 + y^2 \leq 4\}.$$

6. (*12 points*) Evaluate the integral

$$\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x \sqrt{1+\cos^2 x}\, dx\, dy.$$

7. For parts (a)–(c), let $f(x) = \int_2^x \cos(\pi t^2)\,dt$.

   (a) *(5 points)* Find the second Taylor polynomial, $T_2(x)$, for $f(x)$ based at $b = 2$.

   (b) *(3 points)* Use your answer to part (a) to approximate $\int_2^{2.1} \cos(\pi t^2)\,dt$.

   (c) *(6 points)* Find an upper bound on the error for your answer from part (b).

   (Note: For full credit, you do not need to find a tight upper bound, but you must justify your answer with Taylor's inequality.)

8. For this problem, you may use the following basic Taylor series:

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \quad e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad \sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \quad \cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$$

(a) *(6 points)* Find the Taylor series for $f(x) = \dfrac{x^3}{1+x^4} - 3x\sin(x^2)$ based at $b = 0$. Express your answer using $\sum$–notation.

(b) *(3 points)* Find the open interval of convergence for the series you found in (a).

(c) *(5 points)* Find $f^{(2023)}(0)$, i.e. the 2023$^{\text{rd}}$ derivative of f at 0.

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